KTH Computer Science
and Communication

2F1120 Spektrala transformer för Media
Solutions to Steiglitz Chapter 7

Preface

This document contains solutions to selected problems from Ken Steiglitz's book: "A Digital Signal Processing Primer" published by Addison-Wesley. Refer to the book for the problem text.

7.1

From the text book:

$$\langle \mathbf{v}, \mathbf{w} \rangle = v_x w_x + v_y w_y + v_z w_z$$

Where v_x is the component of the vector $\mathbf{v}$ along the x-axis, and so on for the other terms. If we consider only two dimensions for simplicity, and we define θ_v as the angle between the vector $\mathbf{v}$ and the x-axis:

$$\begin{aligned} v_x &= |\mathbf{v}| \cos \theta_v \\ v_y &= |\mathbf{v}| \sin \theta_v \end{aligned}$$

We have then:

$$\begin{aligned} \langle \mathbf{v}, \mathbf{w} \rangle &= v_x w_x + v_y w_y \\ &= |\mathbf{v}| \cos \theta_v |\mathbf{w}| \cos \theta_w + |\mathbf{v}| \sin \theta_v |\mathbf{w}| \sin \theta_w \\ &= |\mathbf{v}||\mathbf{w}|(\cos \theta_v \cos \theta_w + \sin \theta_v \sin \theta_w) \\ &= |\mathbf{v}||\mathbf{w}| \cos(\theta_v - \theta_w) = |\mathbf{v}||\mathbf{w}| \cos \theta_{vw} \end{aligned}$$

As we wanted to prove.

7.2

From geometry we know that three points in a three dimensional space always lie on a plane. Since $\mathbf{v}$ and $\mathbf{w}$ start from the same point (the origin), we can always consider them as lying on a plane and derive the expression for $\langle \mathbf{v}, \mathbf{w} \rangle$ the same way as in the previous example.

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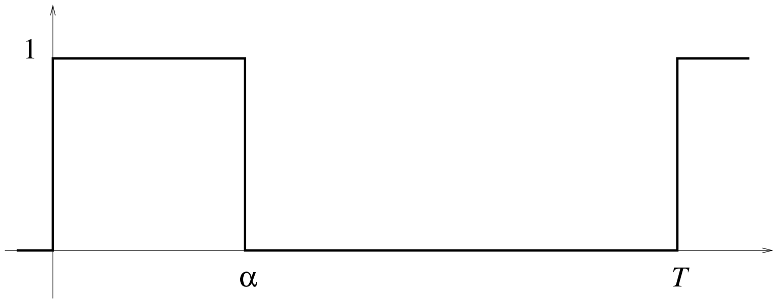

Figure 1.

7.3

From the text book (Equation 3.1):

$$\begin{aligned}
C_n &= \frac{1}{T}\left(\int_0^{T/2} e^{-jn\omega_0 t}dt - \int_{T/2}^{T} e^{-jn\omega_0 t}dt\right)\\
&= \frac{1}{T}\left[\frac{1}{-jn\omega_0}e^{-jn\omega_0 t}\right]_0^{T/2} - \frac{1}{T}\left[\frac{1}{-jn\omega_0}e^{-jn\omega_0 t}\right]_{T/2}^{T}\\
&= \frac{1}{-jn\omega_0 T}\left(e^{-jn\omega_0 T/2}-1\right) - \frac{1}{-jn\omega_0 T}\left(e^{-jn\omega_0 T} - e^{-jn\omega_0 T/2}\right)
\end{aligned}$$

And, imposing $\omega_0 = 2\pi/T$:

$$\begin{aligned}
&= \frac{1}{j2n\pi}\left(1-e^{-jn\pi}\right) + \frac{1}{j2n\pi}\left(e^{-j2n\pi} - e^{-jn\pi}\right)\\
&= \frac{1}{j2n\pi}\left(1-2e^{-jn\pi} + (e^{-jn\pi})^2\right)\\
&= \frac{1}{j2n\pi}\left(1-e^{-jn\pi}\right)^2
\end{aligned}$$

If n is even $e^{-j2k\pi} = 1$, and $C_n = 0, \quad \forall n = 2k$. If n is odd, $e^{-j2k\pi} = -1$, and $C_n = \frac{4}{j2n\pi} = -\frac{2j}{n\pi}, \quad \forall n = 2k-1$.

7.4

The Fourier coefficients are obtained multiplying the signal $y(t)$ with a sinusoidal function $s(t)$ at a certain frequency and then integrating over time. If $s(t)$ is a sinus (an odd function), and the function $y(t)$ is even, then their product $g(t) = y(t)s(t)$ will be odd, that is $g(t) = -g(-t)$. Since the integral extends over time, to each contributions for $t < 0$ corresponds its opposite for $t > 0$. This means that the integral will be zero, and so the coefficient in the Fourier series.

The same reasoning applies in the case $y(t)$ is odd, and $s(t)$ is a cosine.

7.5

The function is depicted in Figure 1. Following the same procedure as in Exercise 7.3 we obtain:

$$\begin{aligned} C_n &= \frac{1}{T}\int_0^{\alpha} e^{-jn\omega_0 t}dt \\ &= \frac{1}{T}\left[\frac{1}{-jn\omega_0}e^{-jn\omega_0 t}\right]_0^{\alpha} \\ &= \frac{1}{-jn\omega_0 T}\left(e^{-jn\omega_0\alpha}-1\right) \end{aligned}$$

And with $\omega_0 = 2\pi/T$:

$$= \frac{j}{2n\pi}\left(e^{-j2n\pi\alpha/T}-1\right)$$

Note that in this case the result is dependent on the ratio between α and T. If $\alpha = T/2$ we obtain the same result as in 7.4.