

Topics in DD2542  
Seminar course in Theoretical Computer Science  
Circuit Complexity  
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**Abstract**

An brief discussion of topics to cover in the course.

## 1 Introduction

This is a seminar course and as such the students are expected to take an active part. Exactly how to organize the course depends on the number of students. Examination is likely to be in the form of homeworks, student lectures, and possibly note-taking.

There is no single book that covers all topics, but one good source is the book by Arora and Barak “Computational Complexity: A Modern Approach”. An electronic version of this book is available on the Internet. For topics not covered by this book we hope to make students in the course take notes (for credit) and make a polished version of these notes available to the entire class.

The aim of the course is to cover some results in concrete complexity. By this we mean explicit lower bounds in various models of computation. Mostly we will be discussing Boolean circuits.

The basic topics will certainly be covered while there will be a choice among the advanced topics. This choice will be discussed with the students taking the course.

## 2 Basic topics

Definitions are needed and we will cover the classical circuit lower bounds from the 1980ies.

- Basic definitions of circuits and formulas. Size, depth, and the concept of monotone circuits. Definition and examples of Boolean functions.

- Relationships of circuit size and depth to Turing machine complexity. Example of circuits. The classes NC and AC.
- Lower bounds for circuit and formula size for random Boolean functions. We will also prove (almost) matching upper bounds.
- The methods for proving lower bounds for formula size of Krapchenko and Nechiporuk.
- Lower bounds for general Boolean circuits of concrete functions.
- Lower bounds for small depth Boolean circuits. Extension to similar circuits containing parity gates.
- Lower bounds for size of monotone circuits computing the clique function.
- Lower bounds for depth of monotone circuits computing st-connectivity. Karchmer-Wigderson games.

### 3 Advanced topics

This is a list of possible topics. Any student taking the class should feel free to propose additional topics.

- Hierarchy theorems for the depth of Boolean circuits. The constant-depth case and the monotone case.
- Lower bounds for small depths circuits computing the clique function.
- The relationship among the following measures. Degree of a Boolean function, decision tree depth, (block) sensitivity and the number of essential variables.
- Circuits of small depth containing threshold gates. Both weighted and un-weighted threshold circuits. Lower bounds and upper bounds.
- Branching programs of constant width, Barrington's theorem.
- Almost cubic lower bounds for formula size of concrete problems inside NP.
- Natural proofs as a way to motivate why it is difficult to prove lower bounds for general circuits.