

## Notation Syllabus

### Vector and Matrices

A sequence of variables  $x_i$  for  $i$  going from 1 to  $n$  can be interpreted as a column vector  $x$  or a row vector  $x^T$  in the corresponding standard basis. The inner product  $\sum_i x_i y_i$  is denoted as  $x^T y$ . The length of a vector is  $\sqrt{x^T x}$  and is denoted as  $|x|$ .

A double indexed set of variables  $a_{ij}$  is interpreted as a matrix  $A$ . We usually denote vectors in small caps and matrices in big caps, but this convention may change to adapt to the context.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad x^T = [x_1 \quad x_2 \quad \cdots \quad x_n] \quad A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

The product between a column vector  $x$  with  $m$  values and a row vector  $y^T$  with  $n$ , is denoted by  $xy^T$  and is a matrix of rank at most 1 with  $m$  rows and  $n$  columns. The product between two matrices  $A$  and  $B$  with  $m$  rows and  $n$  columns is denoted as  $A \bullet B$ .

$$xy^T = \begin{bmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \vdots & \vdots & \ddots & \vdots \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{bmatrix} \quad A \bullet B = \begin{bmatrix} a_{1,1} b_{1,1} & a_{1,2} b_{1,2} & \cdots & a_{1,n} b_{1,n} \\ a_{2,1} b_{2,1} & a_{2,2} b_{2,2} & \cdots & a_{2,n} b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} b_{m,1} & a_{m,2} b_{m,2} & \cdots & a_{m,n} b_{m,n} \end{bmatrix}$$

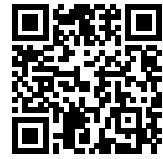
The notations  $x \geq y, x \leq y, x < y, x > y$  for vectors mean that the corresponding inequalities hold pointwise (i.e., if  $x \leq y$  then  $x_i \leq y_i$  for every  $i$ ). We assign a similar meaning to notations  $A \geq B, A \leq B, A < B, A > B$ . The notation  $A \succeq 0$  means that  $A$  is *positive semidefinite*, the notation  $A \succ 0$  that  $A$  is *positive definite*. The notation  $A \succ B$  and  $A \succeq B$  are shortcuts for, respectively,  $A - B \succ 0$  and  $A - B \succeq 0$ .

### Linear programs

We denote linear programs in one of this fashion.

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax \leq b \end{array} \quad \begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax = b \\ & x \geq 0 \end{array}$$

For a linear program such that we denote as  $\mathcal{P}$  the set of feasible solution for that program, and as  $\mathcal{P}_I$  the convex hull of set of integer feasible solutions of the same program, i.e.,  $\mathcal{P}_I = \text{convexhull}(\mathbb{Z}^n \cap \mathcal{P})$ .



*Semidefinite programs*

We denote semidefinite programs (SDP) in one of this way.

maximize $c^T x$ subject to $A_1 \bullet X \leq b_1$ $A_2 \bullet X \leq b_2$ $\vdots$ $A_\ell \bullet X \leq b_\ell$ $X \succeq 0$	maximize $\sum_i c_i (v_0^T v_i)$ subject to $\sum_{i,j} a_{i,j} (v_i^T v_j) \leq b_1$ $\sum_{i,j} a_{i,j}^2 (v_i^T v_j) \leq b_2$ $\vdots$ $\sum_{i,j} a_{i,j}^\ell (v_i^T v_j) \leq b_\ell$ $ v_0  = 1$
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There is no much difference if the constraints are given in form of equations of in form of inequalities. For semidefinite programs such that we denote as  $\mathcal{P}$  the set of feasible solution for that program, and as  $\mathcal{P}_I$  the convex hull of set of integer feasible solutions of the same program, i.e.,  $\mathcal{P}_I = \text{convexhull}(\mathbb{Z}^n \cap \mathcal{P})$ .

*Other notation*

For  $n \leq 0$  and  $k \leq n$ ,

$$\binom{n}{\leq k} = \sum_{i=0}^k \binom{n}{i}$$

For a set  $S$  and integer  $k$ ,

$$\binom{S}{k} = \{T \subseteq S, |T| = k\} \quad \binom{S}{\leq k} = \bigcup_{i=0}^k \binom{S}{i}.$$