Notation Syllabus

Vector and Matrices

A sequence of variables x_i for i going from 1 to n can be interpreted as a column vector x or a row vector x^T in the corresponding standard basis. The inner product $\sum_i x_i y_i$ is denoted as $x^T y$. The length of a vector is $\sqrt{x^T x}$ and is denoted as $|x|$.

A double indexed set of variables a_{ij} is intepreted as a matrix A. We usually denote vectors in small caps and matrices in big caps, but this convention may change to adapt to the context.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad x^T = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \qquad A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ & & \vdots & \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

The product between a column vector x with m values and a row vector y^T with n, is denoted by xy^T and is a matrix of rank at most 1 with m rows and n columns. The product between two matrices A and B with m rows and n columns is denoted as $A \bullet B$.

$$xy^T = \begin{bmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ & & \vdots & \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{bmatrix} \qquad A \bullet B = \begin{bmatrix} a_{1,1} b_{1,1} & a_{1,2} b_{1,2} & \cdots & a_{1,n} b_{1,n} \\ a_{2,1} b_{2,1} & a_{2,2} b_{2,2} & \cdots & a_{2,n} b_{2,n} \\ & & \vdots & \\ a_{m,1} b_{m,1} & a_{m,2} b_{m,2} & \cdots & a_{m,n} b_{m,n} \end{bmatrix}$$

The notations $x \geq y, x \leq y, x < y, x > y$ for vectors mean that the corresponding inequalities hold pointwise (i.e., if $x \leq y$ then $x_i \leq y_i$ for every i). We assign a similar meaning to notations $A \geq B, A \leq B, A < B, A > B$. The notation $A \succeq 0$ means that A is *positive semidefinite*, the notation $A \succ 0$ that A is *positive definite*. The notation $A \succ B$ and $A \succeq B$ are shortcuts for, respectively, $A - B \succ 0$ and A

Linear programs

We denote linear programs in one of this fashion.

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax \leq b \end{array} \qquad \begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax = b \\ & x \geq 0 \end{array}$$

For a linear program such that we denote as $\mathcal{P}$ the set of feasible solution for that program, and as $\mathcal{P}_I$ the convex hull of set of integer feasible solutions of the same program, i.e., $\mathcal{P}_I = \mathsf{convexhull}(\mathbb{Z}^n \cap \mathcal{P})$.

Semidefinite programs

We denote semidefinite programs (SDP) in one of this way.

$$
\begin{aligned}
\text{maximize} \quad & c^T x \\
\text{subject to} \quad & A_1 \bullet X \leq b_1 \\
& A_2 \bullet X \leq b_2 \\
& \vdots \\
& A_\ell \bullet X \leq b_\ell \\
& X \succeq 0
\end{aligned}
\qquad
\begin{aligned}
\text{maximize} \quad & \sum_i c_i (v_0^T v_i) \\
\text{subject to} \quad & \sum_{i,j} a_{i,j} (v_i^T v_j) \leq b_1 \\
& \sum_{i,j} a_{i,j}^2 (v_i^T v_j) \leq b_2 \\
& \vdots \\
& \sum_{i,j} a_{i,j}^\ell (v_i^T v_j) \leq b_\ell \\
& |v_0| = 1
\end{aligned}
$$

There is no much difference if the constraints are given in form of equations of in form of inequalities. For semidefinite programs such that we denote as $\mathcal{P}$ the set of feasible solution for that program, and as $\mathcal{P}_I$ the convex hull of set of integer feasible solutions of the same program, i.e., $\mathcal{P}_I = \mathsf{convexhull}(\mathbb{Z}^n \cap \mathcal{P})$.

Other notation

For $n \leq 0$ and $k \leq n$,

$$\binom{n}{\leq k} = \sum_{i=0}^{k} \binom{n}{i}$$

For a set S and integer k,

$$\binom{S}{k} = \{T \subseteq S, |T| = k\} \qquad \binom{S}{\leq k} = \bigcup_{i=0}^{k} \binom{S}{i}.$$