Proposition 1. *V is a vector space over a field F. Then*

$$\forall v \in V : 0_F \cdot v = 0_V$$

because

$$\begin{array}{rcll} 0_V & = & (0_F v + (-0_F v)) & \textit{(inv)} \\ & = & (0_F + 0_F)v + (-0_F v) & \textit{(add. id)} \\ & = & (0_F v + 0_F v) + (-0_F v) & \textit{(distr)} \\ & = & 0_F v + (0_F v + (-0_F v)) & \textit{(assoc)} \\ & = & 0_F v + 0 & \textit{(inv)} \\ & = & 0_F v & \textit{(add. id)} \end{array}$$

Problems from the book:

Problem 1 V is a vector space over a field F. Find the set of solutions to

$$a \cdot v = 0_V$$

in $F \times V$ for $a \in F$ and $v \in V$. Write *exactly* why each step in the solution is justified as in the proof of (**1**).

Problem 2 $\mathbb{C}^\infty$ is the vector space of all infinite sequences of complex numbers. Prove that the subset H of $\mathbb{C}^\infty$ containing all sequences $v = (a_0, a_1, \ldots)$ with

$$\sum_{j=0}^{\infty} |a_j|^2 \text{ converges}$$

is a vector space.
Hints:
"+" and "·" in $\mathbb{C}^\infty$ are defines as

$$(a_0, a_1 \ldots) + (b_0, b_1, \ldots) = (a_0 + b_0, a_1 + b_1, \ldots)$$

$$\lambda(a_0, a_1, \ldots) = (\lambda a_0, \lambda a_1, \ldots)$$

Every vector space axiom besides *(cl)* follows from the fact that $\mathbb{C}^\infty$ is a vector space.

Problem 3 Prove that the subset C of $\mathbb{C}^\infty$ containing all $v = (a_0, a_1 \ldots)$ such that for any $\epsilon > 0$ there is an integer N with $m, n > N \implies |a_m - a_n| < \epsilon$ is a vector space.
Hint:
For any sequence in C the a_n:s must be getting closer to each other as $n \longrightarrow \infty$. Does this imply anything familiar?