2:nd lecture

We discussed the solutions to the home problems:

Old problem 1 It is in the book

Old problem 2 Use the AG-inequality

Old problem 3 It is equivalent to "v has a limit", search google for *Cauchy condition* or *Caychy sequence*. We gave a proof outline for real sequences. Exercise: deduce the complex case.

All of the theorems below should be familiar from Linear Algebra II.

Everywhere below V is a vector space over a field F. We are (for the moment) only interested in finitedimensional vectors spaces.

Definition 1. *We say that a set of vectors $B = \{b_1, \dots b_n\} \subset V$ is a* ***basis*** *if every $v \in V$ can be written as a* ***finite linear combination*** *of the vectors in B in exactly one way, that is, $v = \sum_{i=1}^{n} m_i \cdot b_i$ has only one solution $(m_1, m_2, \dots, m_n)$.*

Theorem 1. *If V has a basis B_1 and a basis B_2 with B_1 finite, then*

$$|B_1| = |B_2|$$

and thus we can define $\dim V = |B_i|$

Definition 2. *A finite set $M = \{v_1, \dots v_k\}$ of vectors is called* ***linearly independent*** *if*

$$0 = m_1 v_1 + \dots m_k v_k \Longrightarrow m_1 = m_2 = \dots = m_k = 0$$

for $m_i \in F$.

Theorem 2. *if $B = \{b_1, \dots, b_n\} \subset V$ is lin. ind. and $|B| = \dim V$, then B is a basis in B.* [1]

Theorem 3. *Given a set $\{b_1, \dots b_k\}$ of lin. ind. vectors in an $n-$dimensional vector space V, there exist $b_{k+1} \dots b_n$ in V such that $B = \{b_1, \dots b_n\}$ is a basis of V.*

Theorem 4. *If W is a subspace of V,*

$$\dim W \leq \dim V$$

with equality iff $W = V$.

We also proved that the zero vector space does not have a basis (since 0 can be written in two ways as $0 \cdot 0 = 1 \cdot 0$)

We gave examples to spaces, subspaces, dimensions and bases. We proved that $1, (x - a), (x - a)^2, \dots (x - a)^{n-1}$ is a basis for $P = \{p(x) \in F[x] | \deg p < n\}$ and that the coefficients of a polynomial $p(x) = a_0 + a_1 x + \dots + a_{n-1} x^{n-1}$ are $\left(\frac{f(a)}{0!}, \frac{f'(a)}{1!}, \dots \frac{f^{(n-1)}(a)}{(n-1)!} \right)$

provided $\mathrm{char}(F) \geq n$.

[1] Is is basis *in*, basis *of*, basis *for* or something else?