Estimation - 2D Projective Transformations

- Chapter 4 (continue …) -

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Based on Marc Pollefeys' presentations (www.cs.unc.edu/~marc/mvg/slides.html)

Content

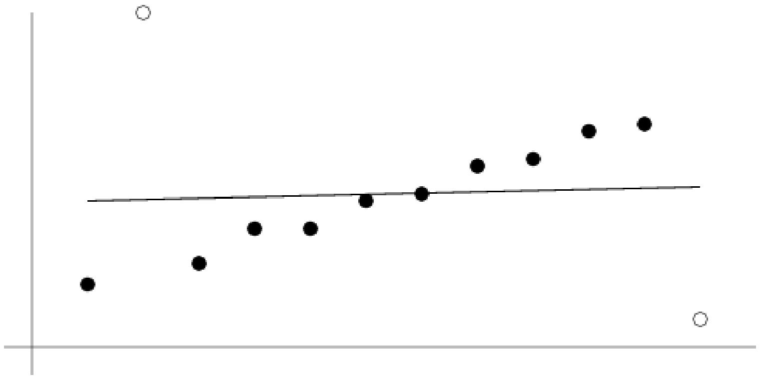
- Chapter 4 continue …
  - Robust Estimation
  - Automatic Computation of a Homography

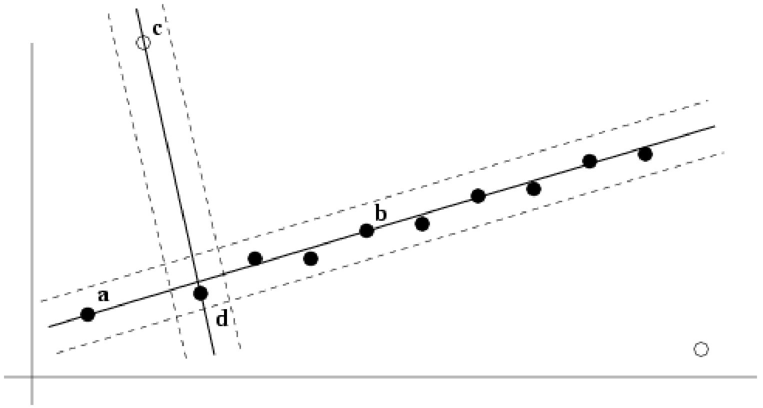
Objective

- Given two images of the same scene
- Compute automatically the homography between them

Robust estimation

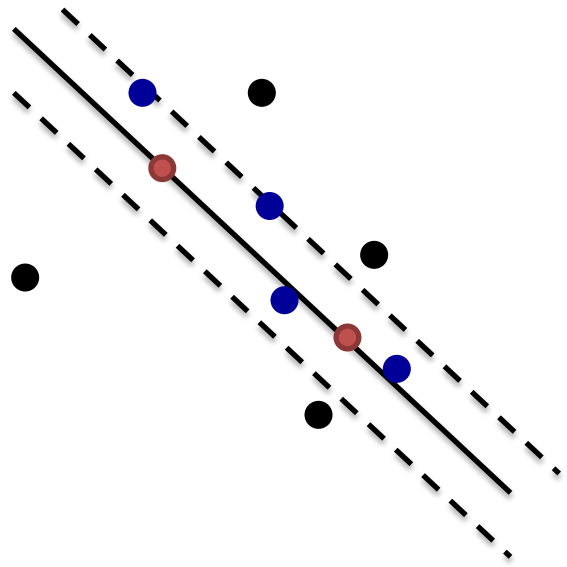
- What if set of matches contains gross outliers?
- **Robust estimation:**
  - Determine a set of inliers from the presented "correspondences" so that the homography estimated in an optimal manner





RANSAC Robust Estimation

- RANSAC: RANdom SAmple Consensus (Fischer and Bolles)
- Given putative correspondences RANSAC determines the set of correct ones
- Idea for line fitting:
  1) choose randomly two points
  2) define a line
  3) determine the support for this line (#points within certain distance)
  1) repeat (1-3) for a certain number of times
  2) choose the line with most support



RANSAC

Objective

Robust fit of a model to a data set S which contains outliers

Algorithm

1) Randomly select a sample of *s* data points from S and instantiate the model from this subset.
2) Determine the set of data points S_i which are within a distance threshold *t* of the model. The set S_i is the consensus set of samples and defines the inliers of S.
3) If the subset of S_i is greater than some threshold *T*, re-estimate the model using all the points in S_i and terminate
4) If the size of S_i is less than *T*, select a new subset and repeat the above.
5) After *N* trials the largest consensus set S_i is selected, and the model is re-estimated using all the points in the subset S_i

RANSAC-What is the distance threshold ?

- Choose distance threshold *t* such that a point is an inlier with a probability α (e.g. 0.95)
- Often empirically
- If measurement error is zero-mean Gaussian noise σ then $d_{\perp}^2$ is sum of squared Gaussian variable and follows distribution χ_m^2 with *m* degree of freedom.

RANSAC -How many samples?

- Choose N (number of samples) to ensure with a probability p that at least one of the random samples of s points is free from outliers
- W is the probability that any selected point is an inlier $e = (1 - W)$ is the probability for an outlier
- At least N selections are required, where $\left(1-(1-e)^s\right)^N = 1 - p$, so that

$$N = \log(1-p) / \log\left(1-(1-e)^s\right)$$

How large is an acceptable consensus set?

- Terminate when inlier ratio reaches expected ratio of inliers,

$$T = (1-e)n$$

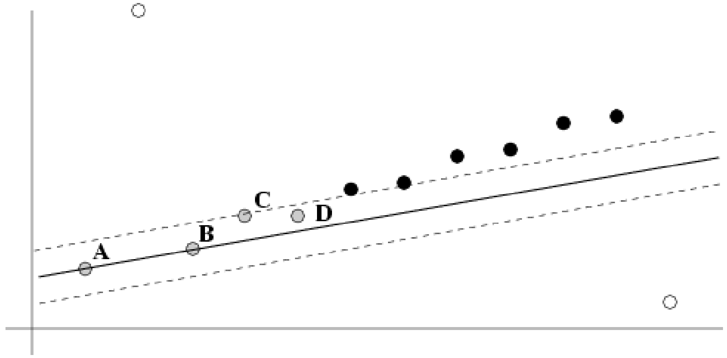
RANSAC -How many samples?

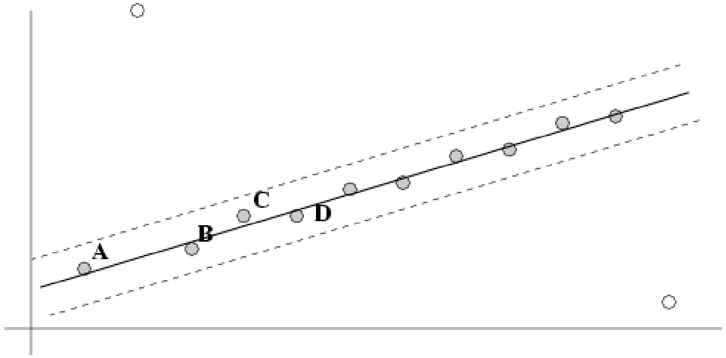
Determining the Number of Samples Adaptively

- Compute number of samples, N, adaptively:
  1. N = ∞, sample_count = 0
  2. While N > sample_count Repeat
     - Choose a sample and count the number of inliers
     - Set e = 1 – (number inliers)/(total number of points)
     - Set N from e and equation of previous slide with p = 0.99
     - Increment the sample_count by 1
  3. Terminate.

Robust Maximum Likelihood Estimation

- Previous MLE algorithm considers fixed set of inliers





Robust cost function: (reclassifies)

$$\mathcal{D} = \sum_{\mathrm{i}} \gamma(d_{\perp i}) \text{ with } \gamma(e) = \begin{cases} e^2 & e^2 < t^2 \text{ inlier} \\ t^2 & e^2 > t^2 \text{ outlier} \end{cases}$$

Automatic Computation of H

Objective:

Compute homography between two images.

Algorithm:

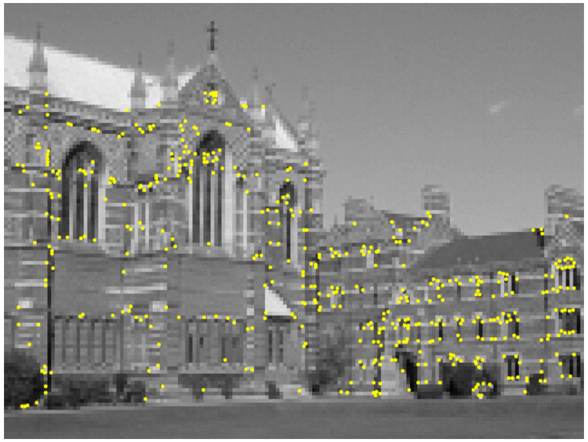
1) **Interest points:** Compute interest points in each image
2) **Putative correspondences:** Compute a set of interest point matches based on some similarity measure
3) **RANSAC robust estimation:** Repeat for *N* samples
   a) Select 4 correspondences and compute H
   b) Calculate the distance $d_\perp$ for each putative match
   c) Compute the number of inliers consistent with H ($d_\perp < t$)

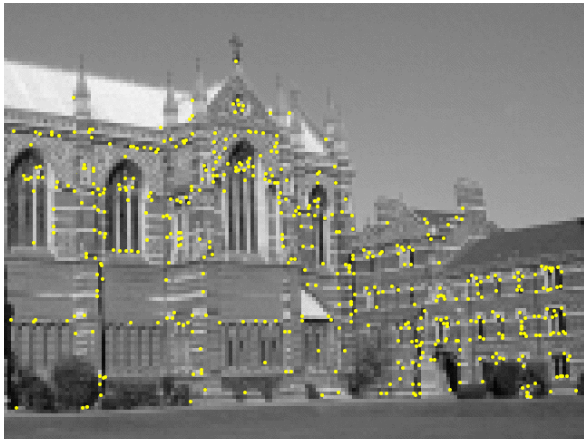
   Choose H with most inliers
4) **Optimal estimation:** re-estimate H from all inliers by minimizing ML cost function,(e.g. with Levenberg-Marquardt).
5) **Guided matching**: Determine more matches using prediction by computed H

Optionally iterate last two steps until convergence

Interest Points

- Automatically compute interest points in each image
- Corner Detection
- Not all points of interest have correspondences in the other image



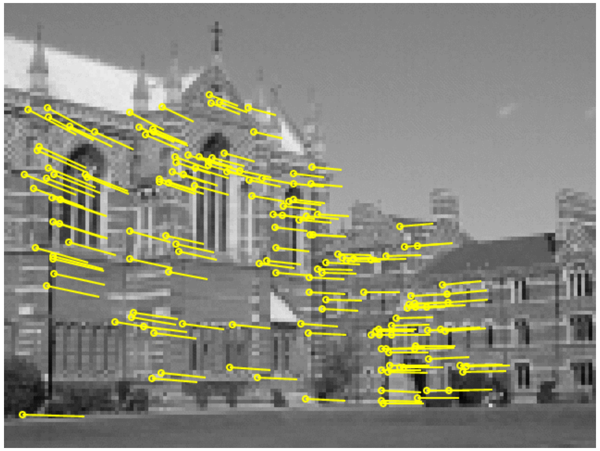


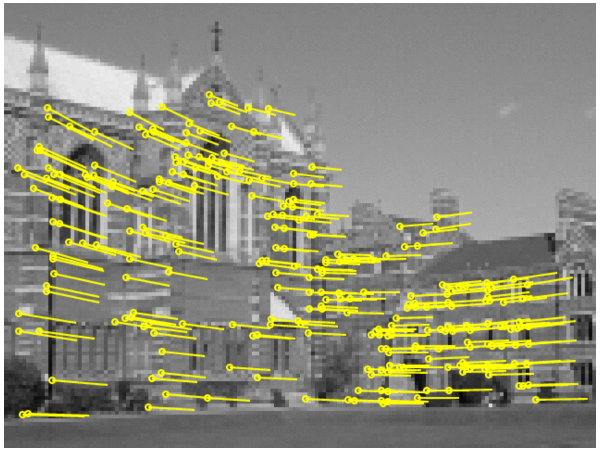
Putative Correspondences

- Compute a set of interest point matches based on some similarity measure
  - Squared sum of intensity differences (SSD)
  - Normalized cross correlation on small neighborhood
- Many correspondences are not the right ones yet

Iteration

- Now we have the “true” correspondences (inliers) by having applied RANSAC
- What’s next?
- **Optimal estimation**: re-estimate H from all inliers by minimizing ML cost function (e.g. with Levenberg-Marquardt)
- **Guided matching**: Use the re-estimated H to determine more matches





Implementation and run details

Number of inliers	1-e	Adaptive N
6	2%	20,038,344
10	3%	2,595,658
44	16%	6,922
58	21%	2,291
73	26%	911
151	56%	43