MULTIPLE VIEW GEOMETRY
CHAPTER 6 - CAMERA MODELS

based on M. Pollefeys' presentation:
http://www.cs.unc.edu/~marc/mvg/course08.ppt

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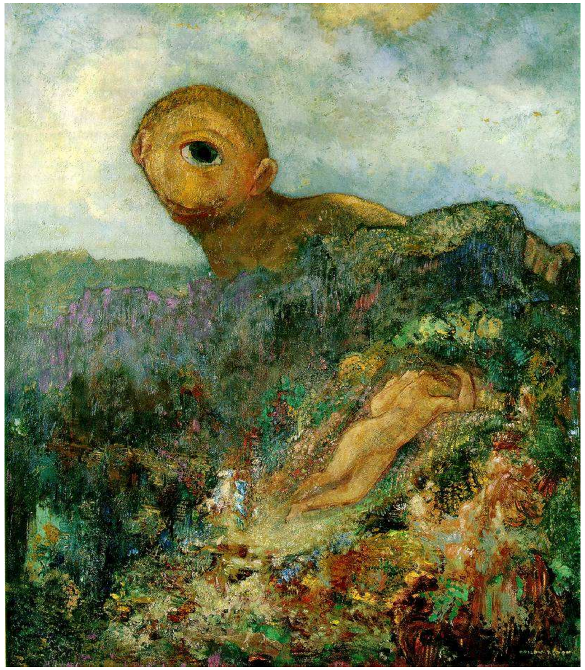
2010/03/11

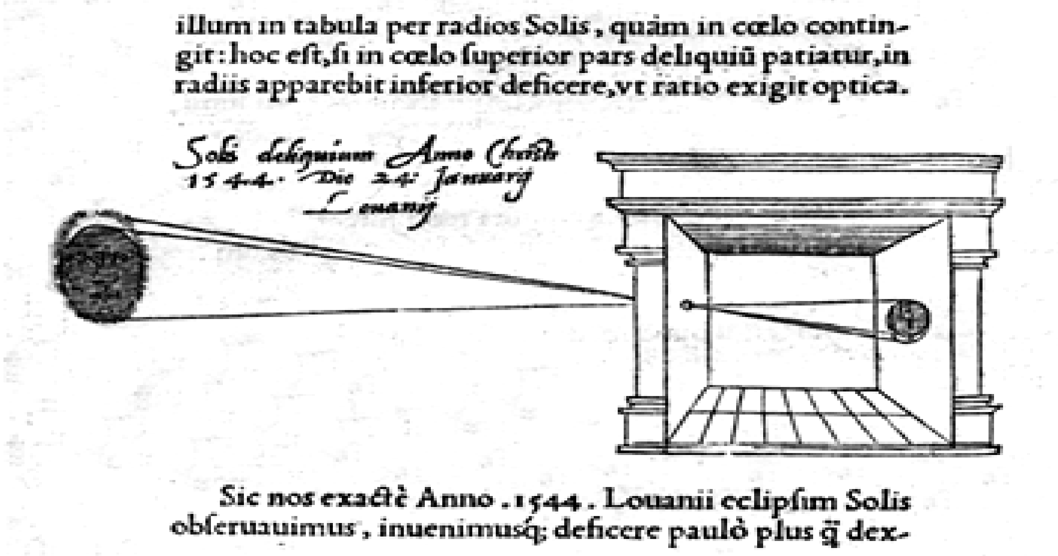
Book Content

1. **Background**: Projective geometry (2D, 3D), Parameter estimation, Algorithm evaluation.
2. **Single View:**
   - Camera model
   - Calibration
   - Single View Geometry



Camera

- Today cameras stems from the ***camera obscura*** - an optical device that projects an image of its surroundings on a screen

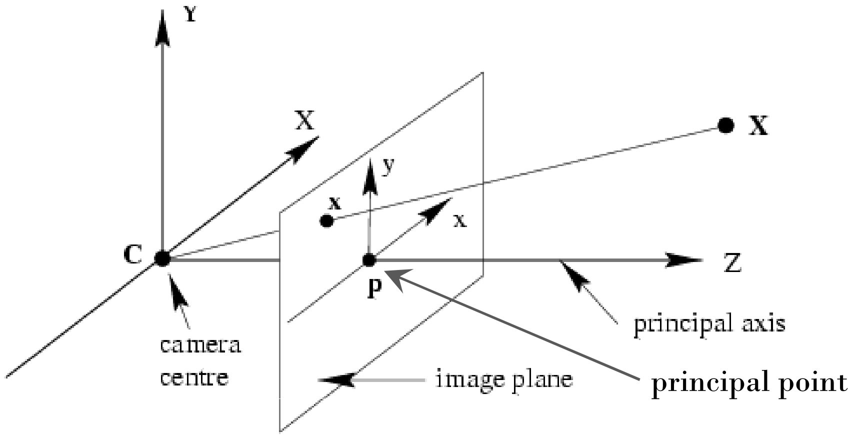


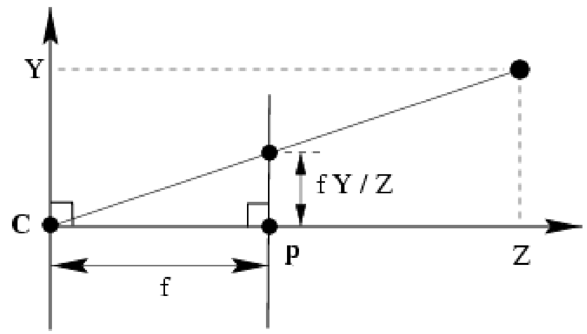
- Camera is a mapping between the 3D world and a 2D image
- In Ch. 6: number of camera models (matrices) that represent 3D-2D mapping

Content Ch.6

1. Finite cameras
2. Properties of projective camera
3. Camera at infinity

Pinhole Camera Model





$$y / f = Y / Z$$

$$y = fY / Z$$

Euclidean coordinates:

$$(X, Y, Z)^T \mapsto (fX / Z, fY / Z)^T$$

Homogeneus coordinates:

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Finite Cameras

Homogeneous Coordinates & Projection

$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Image point: $\mathrm{x} = (\mathrm{fX}, \mathrm{fY}, \mathrm{Z})^{\mathrm{T}}$

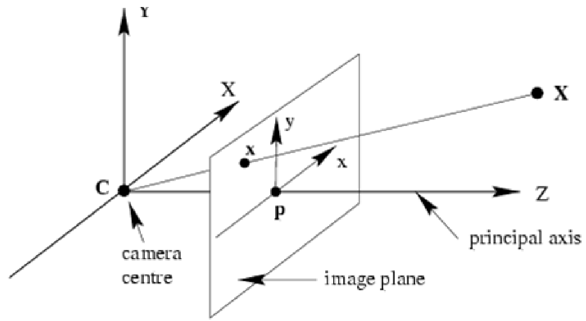
Word point: $\mathbf{X} = (\mathrm{X}, \mathrm{Y}, \mathrm{Z}, 1)^{\mathrm{T}}$

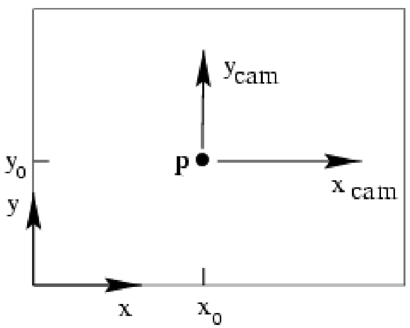
Projection: $\mathrm{x} = \mathrm{P}\mathbf{X}$, where $\mathrm{P} = \mathrm{diag}(f, f, 1)\left[\mathrm{I} \mid 0\right]$

P is 3x4 homogeneous ***camera projection matrix***

Principal Point Offset

- Assumption: image coordinate origin at the principal point $\mathrm{P} = (\mathrm{p}_x, \mathrm{p}_y)^{\mathrm{T}}$
- In practice, it may not be so:





$$(X, Y, Z)^T \mapsto (fX/Z + p_x, fY/Z + p_y)^T$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Projection: $\mathrm{x} = \mathrm{K}[\mathrm{I} \mid 0]\mathrm{X}$ where $K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$

K is ***camera calibration matrix***

Camera Rotation and Translation

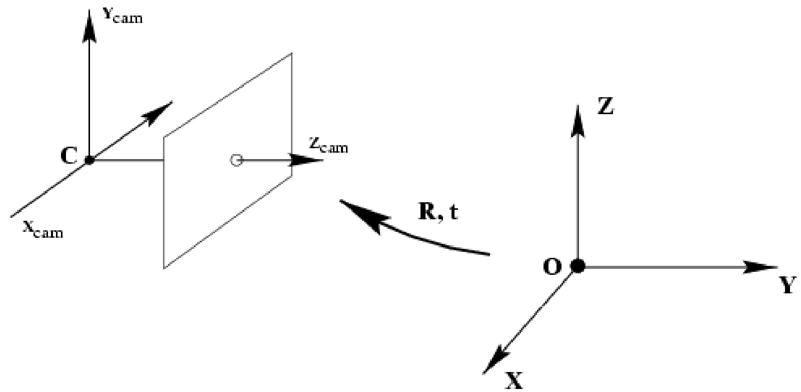
- Word coordinate frame (inhomogeneous 3-vectors):

  $\tilde{X}$ - word point

  $\tilde{X}_{cam}$ - the same point in the camera coord. frame

  $\tilde{C}$ - camera center

  $$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$



- In homogeneous coordinates:

$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} X$$

Projection:

$$\Rightarrow x = \underbrace{KR\left[I \mid -\tilde{C}\right]}_{= P \ (9 \text{ dof})} X$$

- When not making the camera center explicit: $\tilde{X}_{cam} = R\tilde{X} + t$

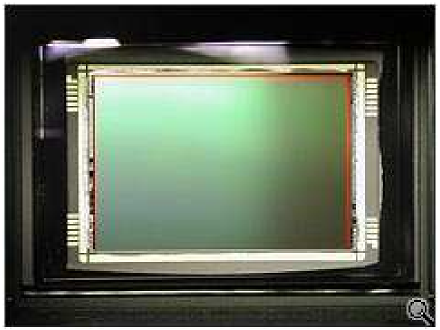
  Projection:

  $$\Rightarrow x = PX \qquad P = K\left[R \mid t\right] \qquad t = -R\tilde{C}$$

CCD Cameras

- Pinhole camera:
  - equal scales in both x- and y-axial directions
  - x-axis perpendicular to y-axis
- CCD cameras may have:
  - non-square pixels
  - not perpendicular x- and y-axis
- Calibration matrix:

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix}$$



Finite Projective Camera: Summary

- Calibration matrix:

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix}$$

- Projection matrix: $P = \underbrace{KR}_{\text{non-singular}} \left[I \mid -\tilde{C} \right]$ 11 dof (5+3+3)

- Representation of P by M:

$$M = KR \qquad \tilde{C} = -M^{-1} p_4$$

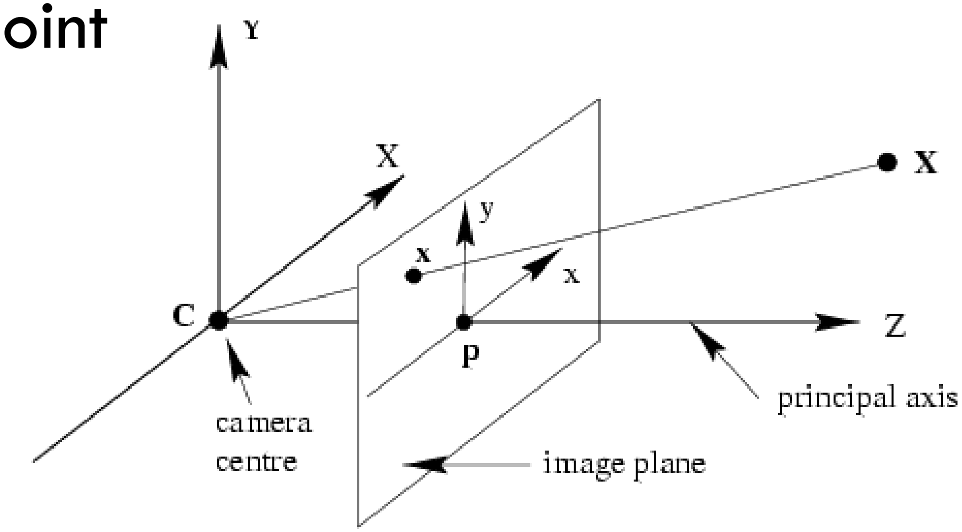
$$\Rightarrow P = M\left[I \mid M^{-1} p_4 \right] \Rightarrow P = \left[M \mid p_4 \right]$$

- Cameras:
  - Finite: $P_{4x3} \mid \det(M) \neq 0$
  - Infinite: If rank P=3, but rank M<3

Projective Camera - Camera Anatomy

$x=PX$

- Camera center
- Column points
- Principal plane
- Axis plane
- Principal point



Projective Camera – Camera Anatomy

Camera Center

- **Projection:** $x = PX$
- **Camera center is a null-space of the camera projection matrix:** $PC = 0$
  - Proof:

$$X = \lambda A + (1-\lambda)Ca$$

$$x = PX = \lambda PA + (1-\lambda)PC = \lambda PA$$

    - For all A all points on AC project on image of A, therefore C is camera center
- **Camera center:**
  - Finite cameras: $C = \begin{pmatrix} -M^{-1}p_4 \\ 1 \end{pmatrix}$
  - Infinite cameras: $C = \begin{pmatrix} d \\ 0 \end{pmatrix}, Md = 0$

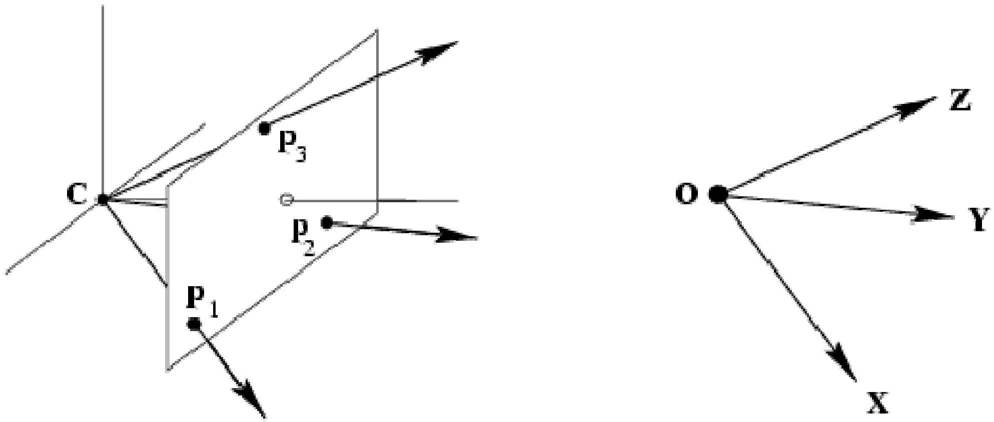
Column Vectors

$$P = [p_1 p_2 p_3 p_4]$$

- Example:

$$[p_2] = [p_1 p_2 p_3 p_4] \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ p_i are image points corresponding to X,Y,Z directions (vanishing points) and origin



Projective Camera – Camera Anatomy

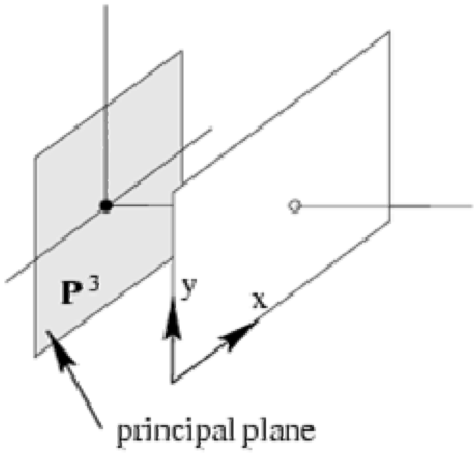
Row Vectors

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathrm{p}^{1\mathsf{T}} \\ \mathrm{p}^{2\mathsf{T}} \\ \mathrm{p}^{3\mathsf{T}} \end{bmatrix}$$

Row vectors p^i can be interpreted as world planes

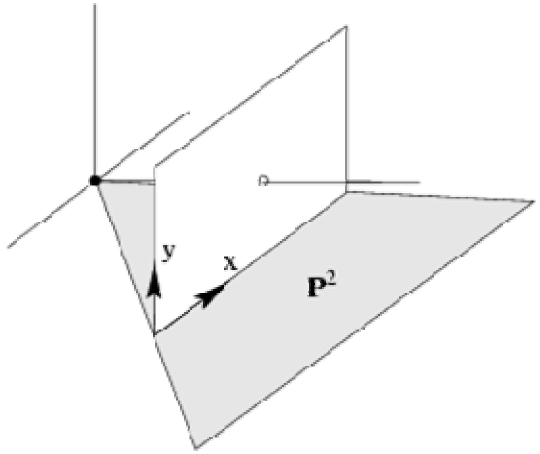
- Principal plane – consists of X which are imaged on the line at infinity of the image

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} \mathrm{p}^{1\mathsf{T}} \\ \mathrm{p}^{2\mathsf{T}} \\ \mathrm{p}^{3\mathsf{T}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



- Axis plane – set of X on P^2 are points on the image x-axis

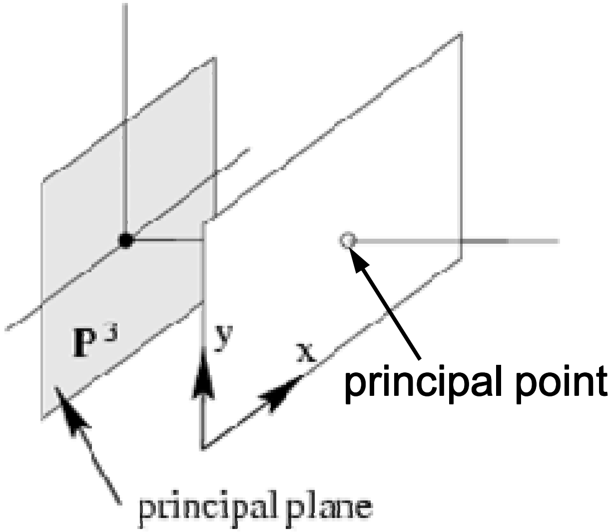
$$\begin{bmatrix} x \\ 0 \\ w \end{bmatrix} = \begin{bmatrix} \mathrm{p}^{1\mathsf{T}} \\ \mathrm{p}^{2\mathsf{T}} \\ \mathrm{p}^{3\mathsf{T}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

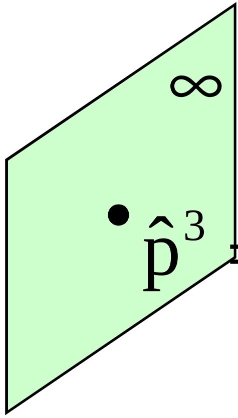


Projective Camera – Camera Anatomy

Principal Point

- Principal axis passes through the camera center C with direction perpendicular to plane P^3; the axis intersect image plan at the *principal point*





$$\hat{\mathrm{p}}^3 = (p_{31}, p_{32}, p_{33}, 0)$$

$$\mathrm{x}_0 = \mathrm{P}\hat{\mathrm{p}}^3 = \mathrm{Mm}^3$$

Action of Projective Camera on Points

- Forward projection: $\mathrm{x} = \mathrm{PX}$
  - Points $D=(d^T,0)^T$ on the plane at infinity represent vanishing points

$$\mathrm{x} = \mathrm{PD} = [\mathrm{M} \mid \mathrm{p}_4]\mathrm{D} = \mathrm{Md}$$

- Back-projection:
  - Intersection of the camera center C and point x (backproj. X)

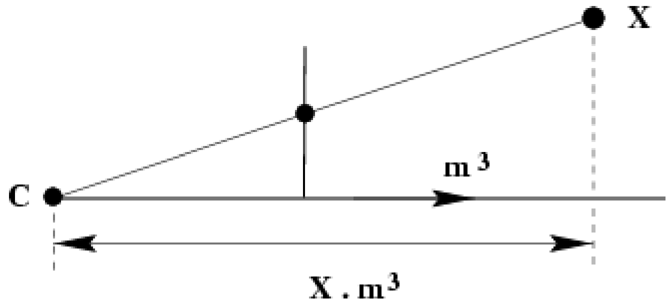
$$\mathrm{PC} = 0$$

$$\mathrm{X} = \mathrm{P}^{+}\mathrm{x} \qquad \mathrm{P}^{+} = \mathrm{P}^{T}\left(\mathrm{PP}^{T}\right)^{-1} \qquad \mathrm{PP}^{+} = \mathrm{I}$$

(pseudo-inverse)

$$\mathrm{X}(\lambda) = \mathrm{P}^{+}\mathrm{x} + \lambda\mathrm{C}$$

Depth of Points



$$w = P^{3^T} X = P^{3^T}(X - C) = m^{3^T}(\tilde{X} - \tilde{C})$$

(PC=0) (dot product)

w - dot product of the ray from C to X with the principal ray direction

If $\det M > 0; \|m^3\| = 1$,
then m^3 unit vector in positive direction

$$\text{depth}(X; P) = \frac{\text{sign}(\det M) w}{T \|m^3\|}$$

$$X = (X, Y, Z, T)^T$$