Camera Calibration

Multiple View Geometry

Based on Marc Pollefeys' Slides

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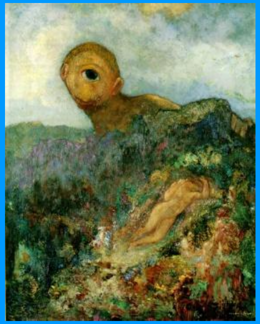
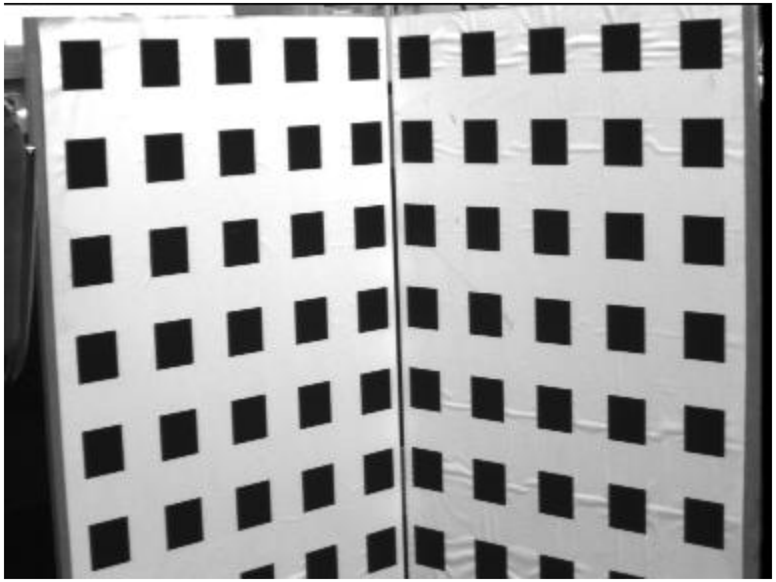
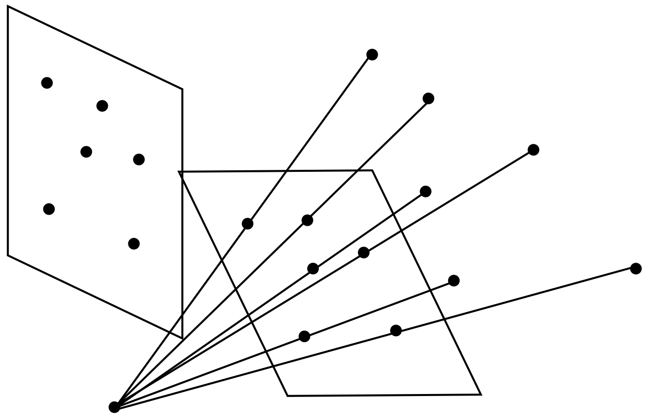


Camera calibration



Resectioning

$$X_i \leftrightarrow x_i \qquad P\,?$$



Basic equations

$$x_i = PX_i$$

$$[x_i]_\times PX_i$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i{}^\top & y_i\mathbf{X}_i{}^\top \\ w_i\mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i{}^\top \\ -y_i\mathbf{X}_i{}^\top & x_i\mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i{}^\top & y_i\mathbf{X}_i{}^\top \\ w_i\mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i{}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$Ap = 0$$

Basic equations

$$\mathrm{Ap} = 0$$

minimal solution

P has 11 dof, 2 independent eq./points

$\Rightarrow$ 5½ correspondences needed (say 6)

Over-determined solution

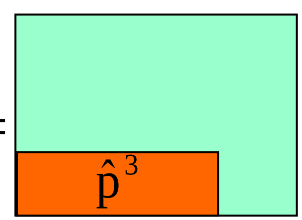
$n \geq 6$ points

minimize $\|\mathrm{Ap}\|$ subject to constraint

$$\|\mathrm{p}\| = 1$$

$$\|\hat{\mathrm{p}}^3\| = 1$$

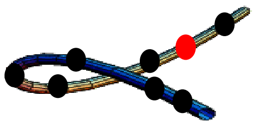
$\mathrm{P} =$



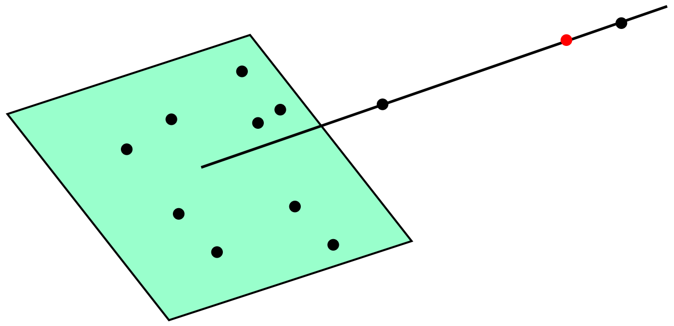
Degenerate configurations

More complicate than 2D case (see Ch.21)

(i) Camera and points on a twisted cubic



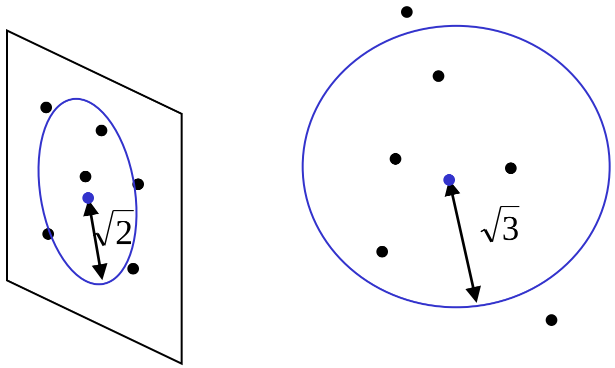
(ii) Points lie on plane or single line passing through projection center



Data normalization

Less obvious

(i) Simple, as before [compact limitation]



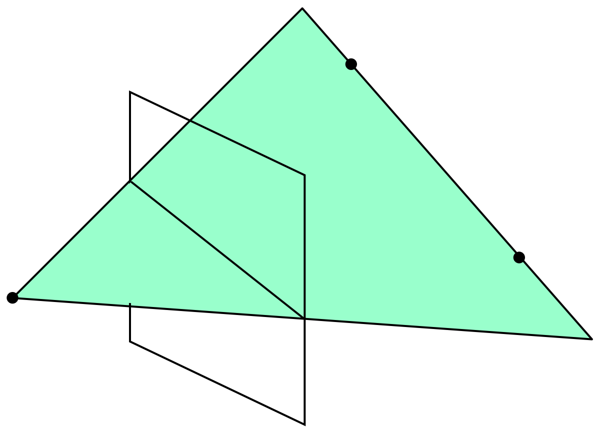
(ii) Anisotropic scaling

Line correspondences

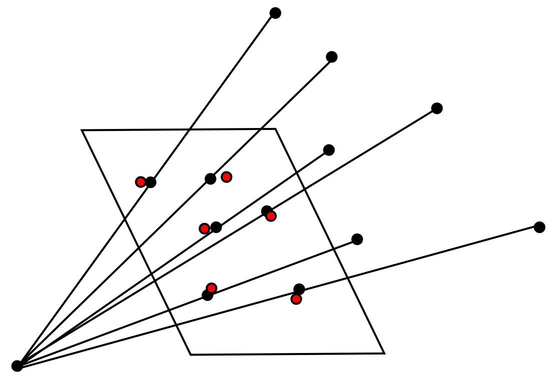
Extend DLT to lines

$$\Pi = \mathrm{P}^{\mathrm{T}}\mathrm{l}_i \qquad \text{(back-project line)}$$

$$\mathrm{l}_i^{\mathrm{T}}\mathrm{P}\mathrm{X}_{1i} \qquad \mathrm{l}_i^{\mathrm{T}}\mathrm{P}\mathrm{X}_{2i} \qquad \text{(2 independent eq.)}$$



Geometric error



$$\sum_i d(\mathbf{x}_i, \hat{\mathbf{x}}_i)^2$$

$$\min_{\mathrm{P}} \sum_i d(\mathbf{x}_i, \mathrm{P}\mathbf{X}_i)^2$$

Gold Standard algorithm

<u>Objective</u>

Given n≥6 3D to 2D point correspondences $\{X_i \leftrightarrow x_i\}$, determine the Maximum Likelihood Estimation of P

<u>Algorithm</u>

(i) Linear solution:

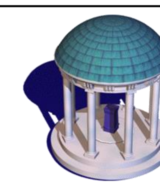
(a) Normalization: $\tilde{X}_i = UX_i \quad \tilde{x}_i = Tx_i$

(b) DLT:

(ii) Minimization of geometric error: using the linear estimate as a starting point minimize the geometric error:

$$\min_{\mathrm{P}} \sum_i d(\tilde{\mathbf{x}}_i, \tilde{\mathrm{P}}\tilde{\mathbf{X}}_i)^2$$

(iii) Denormalization: $P = T^{-1}\tilde{P}U$

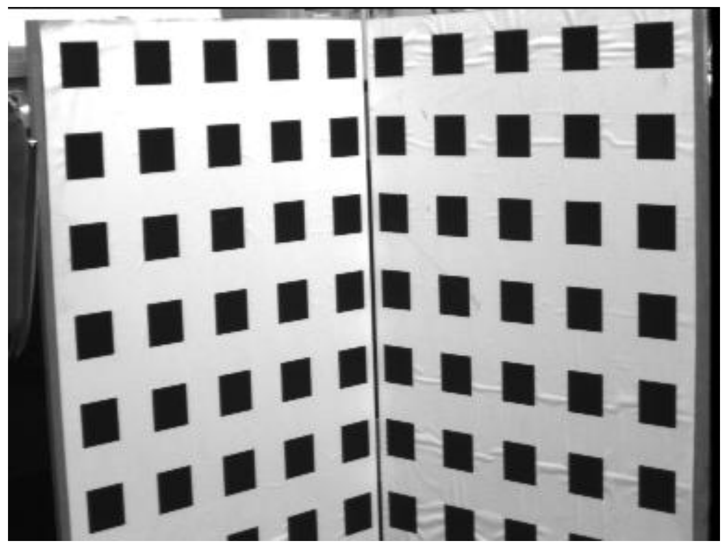


Calibration example

(i) Canny edge detection
(ii) Straight line fitting to the detected edges
(iii) Intersecting the lines to obtain the images corners

typically precision <1/10

(HZ rule of thumb: 5*n* constraints for *n* unknowns



	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364

Errors in the world

$$\sum_i d(\mathbf{X}_i, \widehat{\mathbf{X}}_i)^2$$

$$\mathrm{x}_i = \mathrm{P}\overset{\frown}{\mathrm{X}}_i$$

Errors in the image and in the world

$$\sum_{i=1}^{n} d_{\mathrm{Mah}}(\mathbf{x}_i, \mathrm{P}\widehat{\mathbf{X}}_i)^2 + d_{\mathrm{Mah}}(\mathbf{X}_i, \widehat{\mathbf{X}}_i)^2$$

$$\overset{\frown}{\mathrm{X}}_i$$

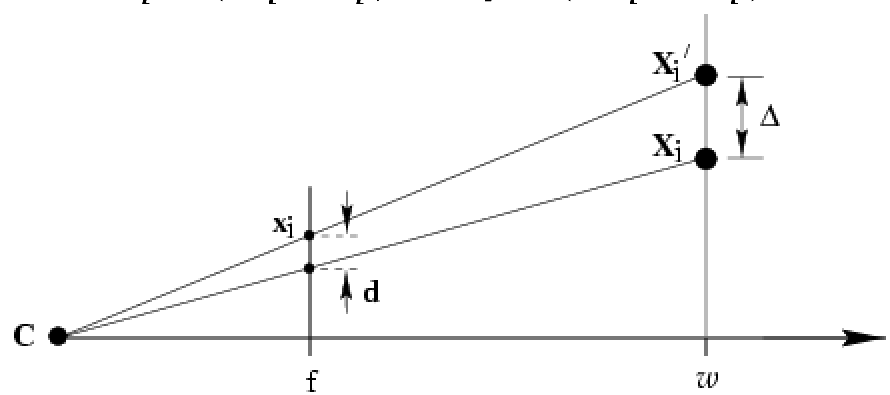
Geometric interpretation of algebraic error

$$\sum_i \left(\hat{w}_i d(\mathrm{x}_i, \hat{\mathrm{x}}_i)\right)^2$$

$$\hat{w}_i\left(\hat{x}_i, \hat{y}_i, 1\right) = \mathrm{PX}_i \qquad \hat{w}_i = \pm\left\|\hat{\mathrm{p}}^3\right\| \mathrm{depth}(\mathrm{X};\mathrm{P})$$

$$\text{therefore, if } \left\|\hat{\mathrm{p}}^3\right\| = 1 \text{ then}$$

$$\hat{w}_i d(\mathrm{x}_i, \hat{\mathrm{x}}_i) \sim f d(\mathrm{X}_i, \hat{\mathrm{X}}_i)$$



note invariance to 2D and 3D similarities given proper normalization

Estimation of affine camera

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i{}^\top & y_i\mathbf{X}_i{}^\top \\ w_i\mathbf{X}_i{}^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i{}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -\mathbf{X}_i{}^\top \\ \mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \end{pmatrix} + \begin{pmatrix} y_i \\ -x_i \end{pmatrix} = \mathbf{0}$$

$$||\mathtt{A}\mathbf{p}||^2 = \sum_i \left(x_i - \mathbf{P}^{1\top}\mathbf{X}_i\right)^2 + \left(y_i - \mathbf{P}^{2\top}\mathbf{X}_i\right)^2 = \sum_i d(\mathbf{x}_i, \hat{\mathbf{x}}_i)^2$$

note that in this case algebraic error = geometric error

Gold Standard algorithm

Objective

Given n≥4 3D to 2D point correspondences $\{X_i \leftrightarrow x_i\}$, determine the Maximum Likelihood Estimation of P

(remember $P^{3T}=(0,0,0,1)$)

Algorithm

(i) **Normalization:** $\tilde{X}_i = UX_i$ $\tilde{x}_i = Tx_i$

(ii) For each correspondence

$$\begin{bmatrix} \mathbf{0}^\top & -\mathbf{X}_i{}^\top \\ \mathbf{X}_i{}^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \end{pmatrix} + \begin{pmatrix} y_i \\ -x_i \end{pmatrix} = \mathbf{0}$$

$$A_8 p_8 = b$$

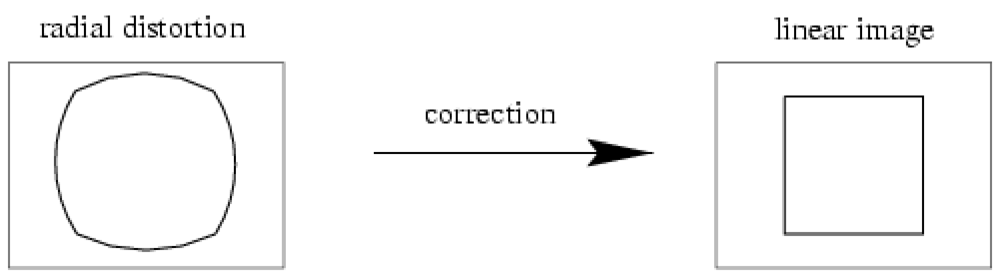
(iii) solution is

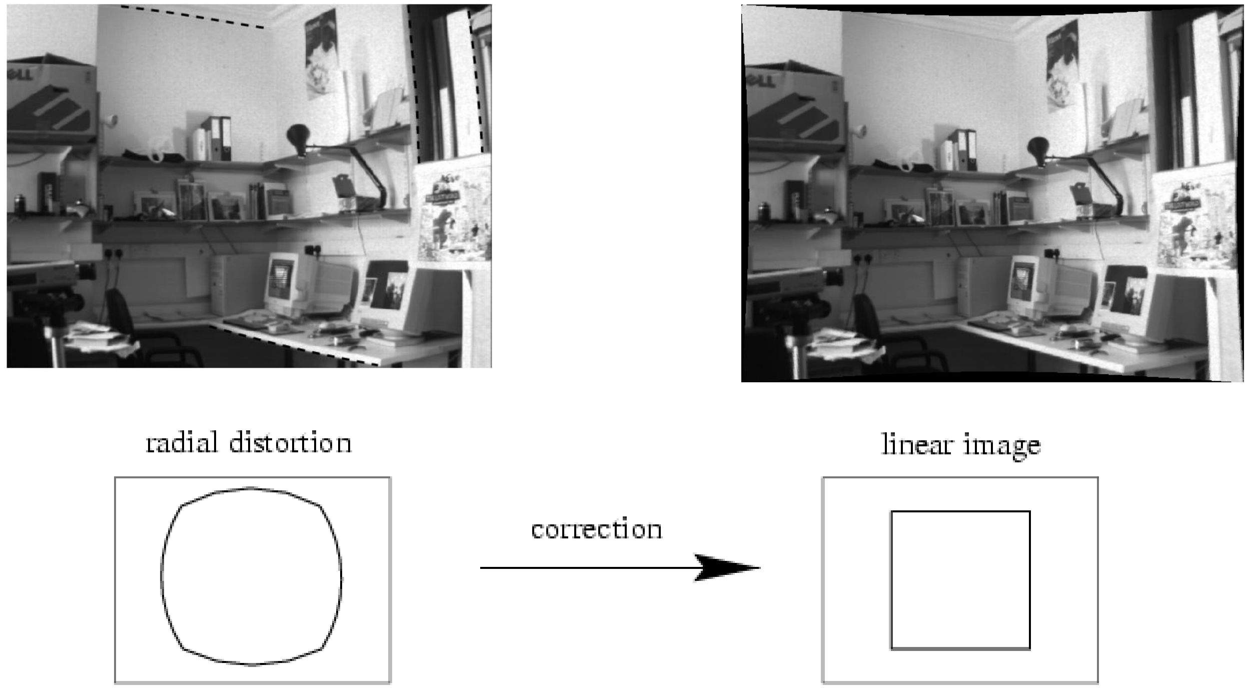
$$p_8 = A_8{}^+ b$$

(iv) **Denormalization:** $P = T^{-1}\tilde{P}U$

Radial distortion

Short(cheaper) and long focal length





$$(\tilde{x}, \tilde{y}, 1)^\top = [\mathtt{I} \mid \mathbf{0}]\mathbf{X}_{\mathrm{cam}}$$

$$\begin{pmatrix} x_d \\ y_d \end{pmatrix} = L(\tilde{r}) \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$$

Correction of distortion

$$\hat{x} = x_c + L(r)(x - x_c) \qquad \hat{y} = y_c + L(r)(y - y_c)$$

Choice of the distortion function and center

$$r^2 = (x - x_c)^2 + (y - y_c)^2$$

$$L(r) = 1 + k_1 r + k_2 r^2 + k_3 r^3 + \dots$$

$$\{k_1, k_2, \dots, x_c, y_c\}$$

Computing the parameters of the distortion function

(i) Define a cost function for deviating from linear projections

(ii) Iteratively Minimize with additional unknowns

(iii) [Warping]?

An example

	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1580.5	1.0044	0.75	377.53	299.12	0.179
iterative	1580.7	1.0044	0.70	377.42	299.02	0.179
algebraic	1556.0	1.0000	0.00	372.42	291.86	0.381
iterative	1556.6	1.0000	0.00	372.41	291.86	0.380
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364
algebraic	1633.4	1.0000	0.00	371.21	293.63	0.601
iterative	1637.2	1.0000	0.00	371.32	293.69	0.601

Top: With radial distortion correction,
bottom: Without radial distortion correction