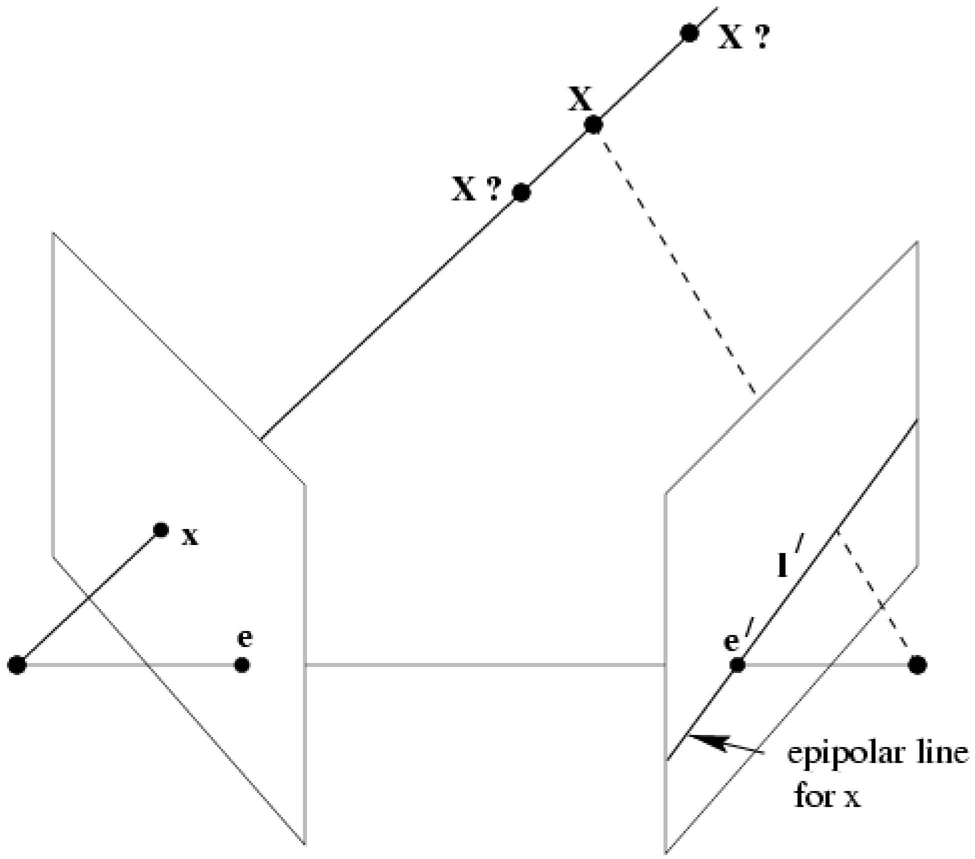
Recap: The epipolar geometry



The fundamental matrix F

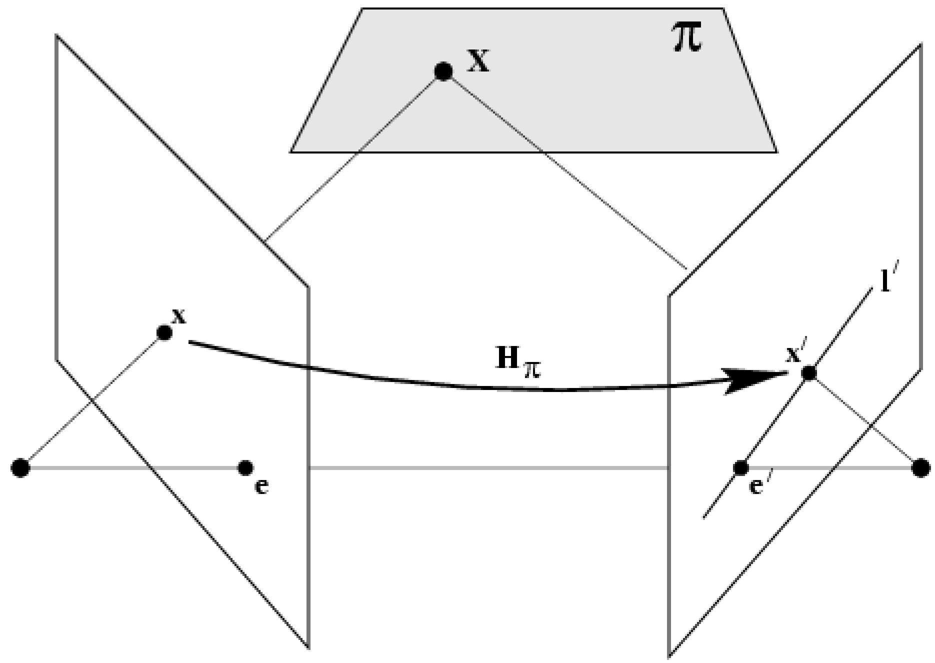
algebraic representation of epipolar geometry

$$\mathrm{x} \mapsto \mathrm{l}'$$

we will see that mapping is (singular) correlation
(i.e. projective mapping from points to lines)
represented by the fundamental matrix F

The fundamental matrix F

geometric derivation



$$x' = H_{\pi} x$$

$$l' = e' \times x' = [e']_{\times} H_{\pi} x = Fx$$

mapping from 2-D to 1-D family (rank 2)

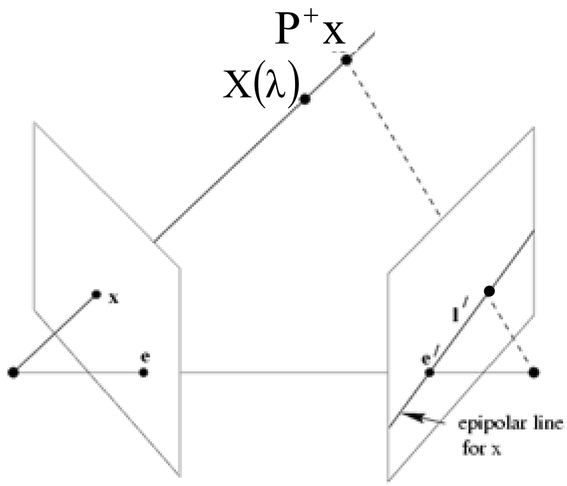
The fundamental matrix F

algebraic derivation

$$X(\lambda) = P^{+}x + \lambda C \qquad \left(P^{+}P = I\right)$$

$$l = P'C \times P'P^{+}x$$

$$F = [e']_{\times} P'P^{+}$$



(note: doesn’t work for C=C’ $\Rightarrow$ F=0)

From F to the Cameras – Some Useful Properties of F

(i) Projective Invariance

(ii) Projective Ambiguity

(iii) Canonical Cameras given F

Projective transformation and invariance

Derivation based purely on projective concepts

$$\hat{x} = Hx,\ \hat{x}' = H'x' \Rightarrow \hat{F} = H'^{-T} F H^{-1}$$

F invariant to transformations of projective 3-space

$$x = PX = (PH)(H^{-1}X) = \hat{P}\hat{X}$$
$$x' = P'X = (P'H)(H^{-1}X) = \hat{P}'\hat{X}$$

$(P, P') \mapsto F$ unique

$F \mapsto (P, P')$ not unique

canonical form

$$P = [I \mid 0] \qquad P' = [M \mid m] \qquad F = [m]_\times M$$

Projective ambiguity of cameras given F

previous slide: at least projective ambiguity
this slide: not more! ???? (this slide: it is the only ambiguity)

Show that if F is same for (P,P') and ($\tilde{P}$,$\tilde{P}$'),
there exists a projective transformation H so that
$\tilde{P}$=PH and $\tilde{P}$'=P'H

$$P = [I \mid 0] \quad P' = [A \mid a] \quad \tilde{P} = [I \mid 0] \quad \tilde{P}' = [\tilde{A} \mid \tilde{a}]$$

$$F = [a]_\times A = [\tilde{a}]_\times \tilde{A}$$

lemma: $$\tilde{a} = ka \qquad \tilde{A} = k^{-1}\left(A + av^T\right)$$

$$aF = a[a]_\times A = 0 = \tilde{a}F \overset{\text{rank 2}}{\Longrightarrow} \tilde{a} = ka$$

$$[a]_\times A = [\tilde{a}]_\times \tilde{A} \Rightarrow [a]_\times\left(k\tilde{A} - A\right) = 0 \Rightarrow \left(k\tilde{A} - A\right) = av^T$$

$$H = \begin{bmatrix} k^{-1}I & 0 \\ k^{-1}v^T & k \end{bmatrix}$$

$$P'H = [A \mid a]\begin{bmatrix} k^{-1}I & 0 \\ k^{-1}v^T & k \end{bmatrix} = [k^{-1}\left(A - av^T\right) \mid ka] = \tilde{P}'$$

(22-15=7, ok)

Canonical cameras given F

F matrix corresponds to P,P' iff P'^{T}FP is skew-symmetric.
This is equivalent to $\left(X^T P'^T FPX = 0, \forall X\right)$

F matrix, S skew-symmetric matrix

$$P = [I \mid 0] \quad P' = [SF \mid e']$$ (fund.matrix=F) (Epipole = e)

$$\left([SF \mid e']^T F [I \mid 0] = \begin{bmatrix} F^T S^T F & 0 \\ e'^T F & 0 \end{bmatrix} = \begin{bmatrix} F^T S^T F & 0 \\ 0 & 0 \end{bmatrix} \right)$$

Possible choice:

$$P = [I \mid 0] \quad P' = [[e']_\times F \mid e']$$

Canonical representation:

$$P = [I \mid 0] \quad P' = [[e']_\times F + e' v^T \mid \lambda e']$$

The essential matrix

~fundamental matrix for calibrated cameras (remove K)

$$E = [t]_\times R = R[R^T t]_\times$$

$$\hat{x}'^T E \hat{x} = 0 \qquad (\hat{x} = K^{-1}x;\ \hat{x}' = K^{-1}x')$$

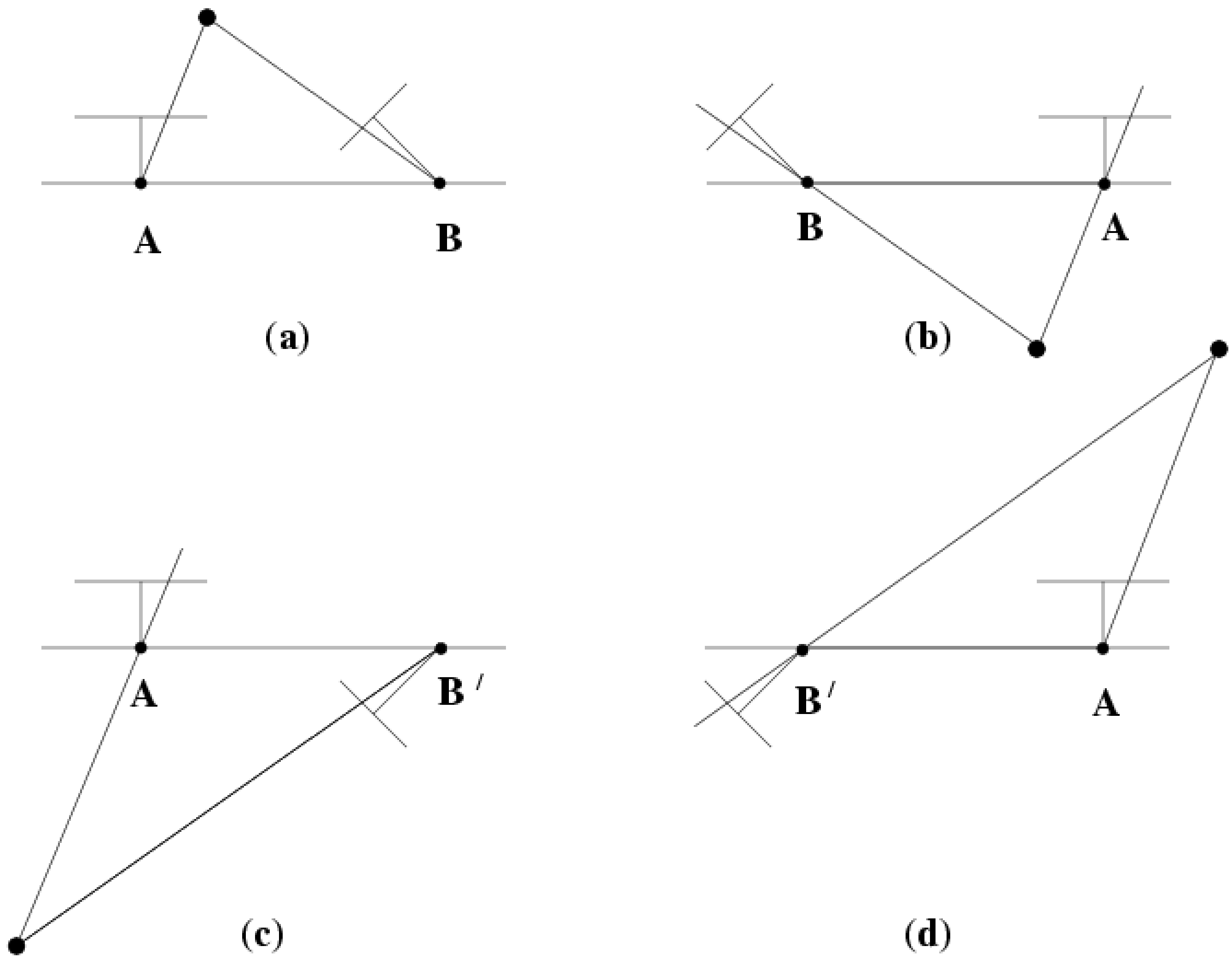
$$E = K'^T F K$$

5 d.o.f. (3 for R; 2 for t up to scale)

E is essential matrix if and only if
two singularvalues are equal (and third=0)

$$E = U\text{diag}(1,1,0)V^T$$

Four possible reconstructions from E



(only one solution where points is in front of both cameras)

3D reconstruction of cameras and structure

reconstruction problem:

given $x_i \leftrightarrow x'_i$, compute P,P' and X_i

$$\mathrm{x}_i = \mathrm{P}\mathrm{X}_i \quad \mathrm{x}'_i = \mathrm{P}\mathrm{X}'_i \qquad \text{for all } i$$

without additional informastion possible
up to projective ambiguity

Outline of reconstruction

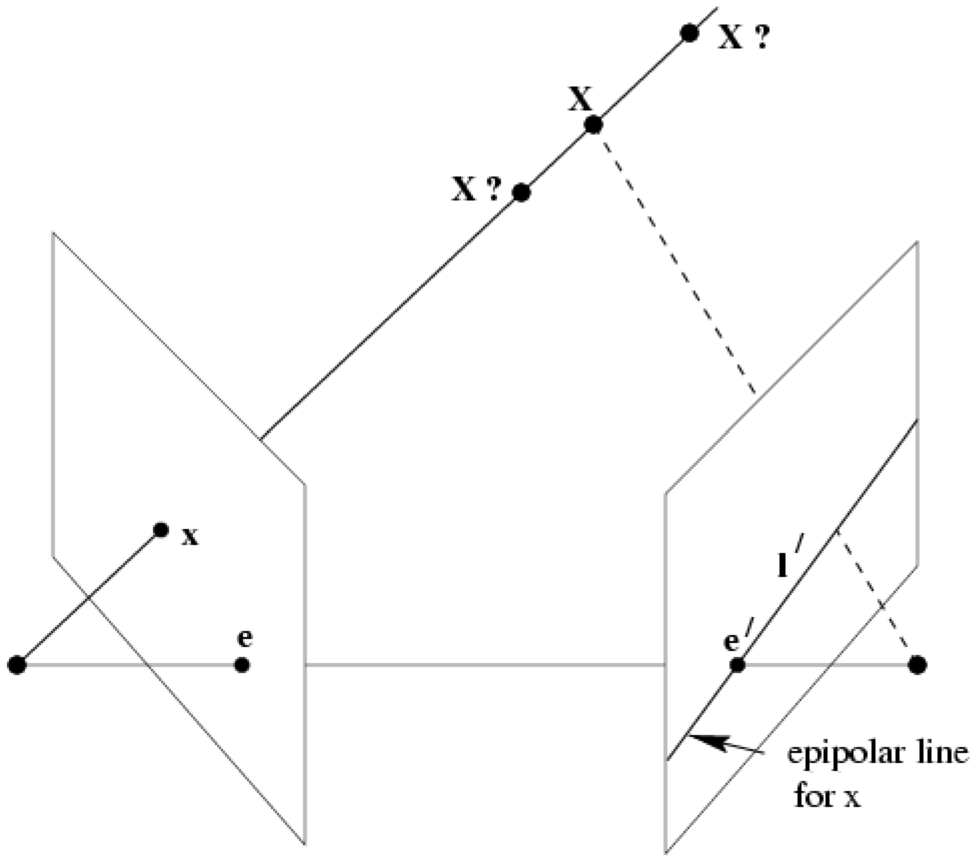
(i) Compute F from correspondences
(ii) Compute camera matrices from F -> Previous Slides
(iii) Compute 3D point for each pair of corresponding points

computation of F
use $x'_i F x_i = 0$ equations, linear in coeff. F
8 points (linear), 7 points (non-linear), 8+ (least-squares)
(more on this next chapter)

computation of camera matrices
use $P = [I \mid 0] \quad P' = [[e']_\times F \mid e']$

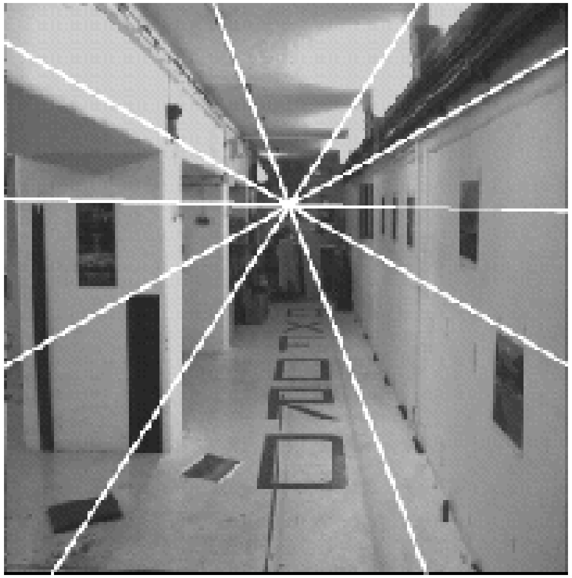
triangulation
compute intersection of two backprojected rays

The epipolar geometry



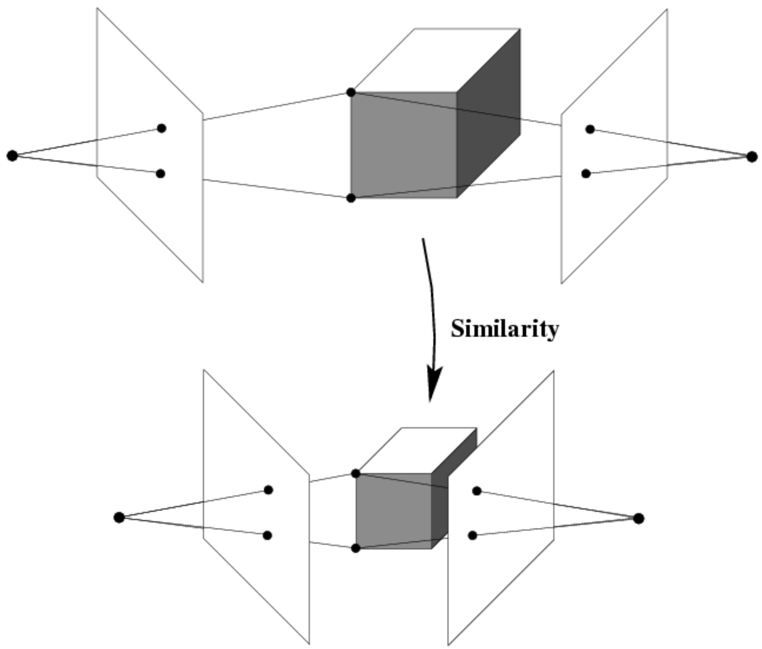
What if only C,C',x are known?

Reconstruction Ambiguity - Scale and Absolute Position and Orientation



- Could be in a puppet house
- East-West or North-South
- Could be a corridor anywhere

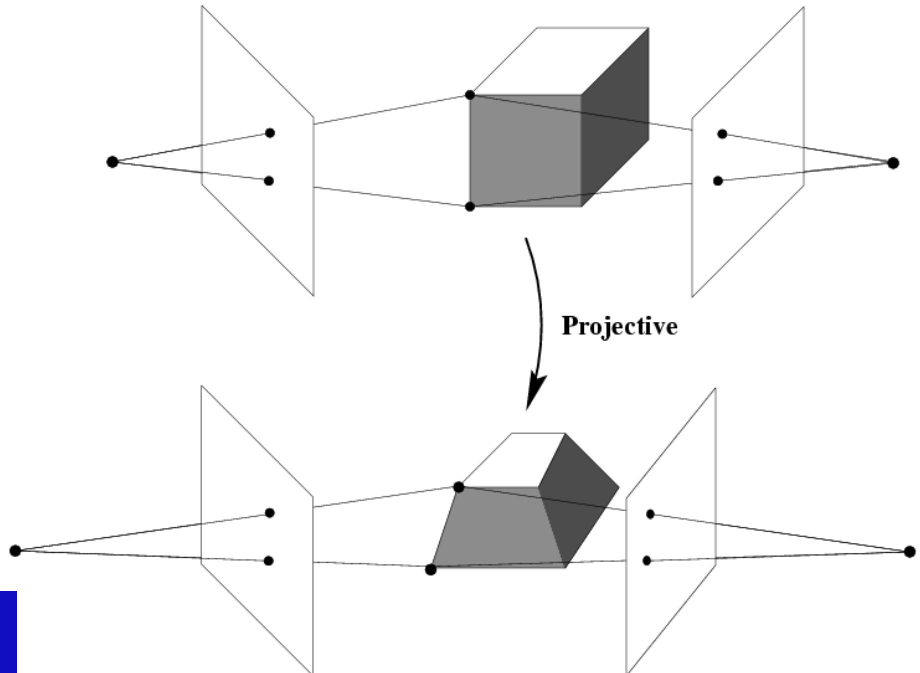
Reconstruction Ambiguity (Calibrated Cameras -> K known): Similarity



$$x_i = PX_i = \left(PH_S^{-1}\right)\left(H_S X_i\right)$$

$$PH_S^{-1} = K[R\,|\,t]\begin{bmatrix} R'^T & -R'^T t' \\ 0 & \lambda \end{bmatrix} = K[RR'^T \,|\, -RR'^T t' + \lambda t]$$

Reconstruction ambiguity (Uncalibrated Camera): Projective



$$x_i = PX_i = \left(PH_P^{-1}\right)\left(H_P X_i\right)$$

Terminology

$x_i \leftrightarrow x'_i$

Original scene X_i

Projective, affine, similarity reconstruction
= reconstruction that is identical to original up to projective, affine, similarity transformation

Literature: Metric and Euclidean reconstruction
= similarity reconstruction = angles and rations between lines can be measured

The projective reconstruction theorem

If a <u>set of point correspondences</u> in two views <u>determine the fundamental matrix uniquely</u>, then the <u>scene and cameras</u> may be reconstructed from these correspondences alone, and any two such reconstructions from these correspondences are <u>projectively equivalent</u>

$$\mathrm{x}_i \leftrightarrow \mathrm{x}'_i \qquad \left(\mathrm{P}_1, \mathrm{P}_1', \{\mathrm{X}_{1i}\}\right) \quad \left(\mathrm{P}_2, \mathrm{P}_2', \{\mathrm{X}_{2i}\}\right)$$

$$\mathrm{P}_2 = \mathrm{P}_1\mathrm{H}^{-1} \quad \mathrm{P}'_2 = \mathrm{P}'_1\mathrm{H}^{-1} \quad \mathrm{X}_{2\mathrm{i}} = \mathrm{H}\mathrm{X}_{1\mathrm{i}} \qquad \left(\mathrm{except}: \mathrm{F}\mathrm{x}_i = \mathrm{x}'_i\mathrm{F} = 0\right)$$

theorem from last class

$$\mathrm{P}_2\left(\mathrm{H}\mathrm{X}_{1i}\right) = \mathrm{P}_1\mathrm{H}^{-1}\mathrm{H}\mathrm{X}_{1i} = \mathrm{P}_1\mathrm{X}_{1i} = \mathrm{x}_i = \mathrm{P}_2\mathrm{X}_{2i}$$

$\Rightarrow$ along same ray of P_2, idem for P'_2

two possibilities: X_{2i}=$\mathrm{H}\mathrm{X}_{1i}$, or points along baseline

<u>key result:</u>
allows reconstruction from pair of uncalibrated images