Camera Auto-calibration

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Auto-calibration

- Determine internal camera parameters from images
- Assumptions:
  - Rigid-body camera movement,: rotation + translation
  - Known point correspondences
  - Fixed internal parameters

Auto-calibration

- $P_i = K_i [R_i \mid t_i]$
- $P_i H = K R_i [I \mid t_i]$ same calibration matrix for each camera
- Sufficiently enough views to constraint H to achieve a metric reconstruction

Auto-calibration

- Use point correspondences to obtain a projective reconstruction
- Recall: xFx' = 0, for x,x' image coordinates, F can be found uniquely.
- Get P and P' from F as:
  - P = [I | 0] and P' = [[e'$_x$ F | e'] (result 9.14)
- Projective reconstruction is accurate up to an homography, why is that?

Auto-calibration

- Projective reconstruction ambiguity
- Given F , let (P_1, P'_1, X_1) and (P_2, P'_2, X_2) two reconstructions

  then there exists a matrix H with $P_2 = P_1 H^{-1}$ and $X_1 = HX_2$.

  (change-of-basis transformation)
- For points:
  - $P_2 X_2 = P_2 (HX1) = P_1 H^{-1} HX_1 = P_1 X_1$
- For camera matrices:
  - Main idea: write F in two different ways and see how they are related → carry this result to writing P's in terms of F (theorem 9.10) turns out they are projectively related

Auto-calibration

- So far: from point correspondences to reconstruction up to homography H.
- $P^i_{true} = P^i H$, obtain H to get K and camera position.
- Use auto-calibration constraints on K

Auto-calibration

- Looking for H in $P^i_{true} = P^i H$
- How many unknowns and their origin ?
- Select first camera as origin then:
  - $P^I_{true} = K[\ I \mid 0]$ & use canonical for $P^I = [\ I \mid 0]$
  - Take $H = [\ A\ t\ ;\ \mathbf{v}^T\ k]$
- Then $K^I\ [I \mid 0\ ] = [\ I \mid 0]\ H$

- H is of the form: $[\,K^{1}\ 0\ ;\ -p^{T}K^{1}\ \ 1\,]$
  - $p = -\,(K^{1})^{-T}\mathbf{v}$
- There are 8 unknowns
  - Related = absolute plane + absolute conic
  - We can only reconstruct up to similarity
    - (15 – 12) + (12 – 7) dof's

Auto-calibration

- Take $P^i = [\, A^i \mid a^i]$,

  $H = [\, K^1 \; 0 \; ; -p^T K^1 \;\; 1]$
- Write $P^i = K^i R^i = (A^i - a^i p^T) K^1$
- Get rid of R^i :
- $R^i = (K^i)^{-1} (A^i - a^i p^T) K^1$ (using $RR^T = I$)
- $K^i K^{iT} ((A^i - a^i p^T) K^1)^{-1} = (A^i - a^i p^T) K^1$

- $K^iK^{iT} = (A^i - a^ip^T)K^1K^{1T}(A^i - a^ip^T)$
- K^iK^{iT} is the dual image of absolute conic ω_∞^*, i.e. mapping of absolute conic under a point homography (hence duality)

$$\omega_\infty^{*i} = (A^i - a^ip^T)\ \omega_\infty^{*1}\ (A^i - a^ip^T)^T$$

$$\omega_\infty^{i} = (A^i - a^ip^T)^{-T}\ \omega_\infty^{1}\ (A^i - a^ip^T)^{-1}$$

- Why is $K^i K^{iT} = \omega^*$?
- Point homography: $x = HX$
- Absolute conic: $\Omega_\infty = C_\infty = I$
- $\omega = H^{-T} C_\infty H^{-1}$, image of absolute conic
- $(KR)^{-T} C_\infty (KR)^{-1} = (KK^T)^{-1}$

- Given a camera matrix P:
- $\omega_\infty^* = P\, Q_\infty^*\, P^T \rightarrow KK^T = P\, Q_\infty^* P^T$
- Why is Q_∞^* important ?
- $C^* = PQ^*P^T$
- ω_∞^* is the outline of the quadric Q_∞^*
- For a general Q this is a conic

Absolute Quadratic

- [I 0 ; 0 0] under Euclidean
- [Q 0 ; 0 0] under Affine
- Arbitrary 4x4 rank 3 matrix under projective
- Behaves like a point under transformations.
- For normal planes Qp gives it's Euclidean direction
  - It's null vector is p_∞

Photosynth