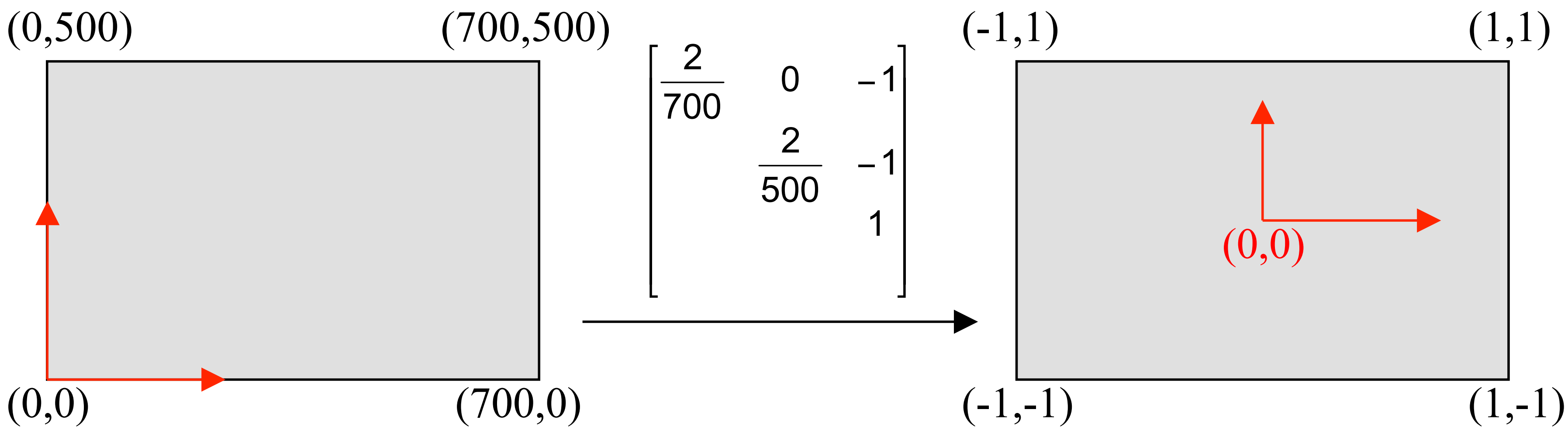
The normalized 8-point algorithm:
Step 1 – Normalize image coordinates

Transform image to ~[-1,1]x[-1,1]



$$\begin{bmatrix} \frac{2}{700} & 0 & -1 \\ & \frac{2}{500} & -1 \\ & & 1 \end{bmatrix}$$

Or normalize coordinates to have mean (0,0) and average norm sqrt(2)

The normalized 8-point algorithm: Step 2 – Compute F

$$\mathrm{x}'^{\mathrm{T}}\,\mathrm{F}_{\mathrm{norm}}\,\mathrm{x} = 0$$

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

$$\begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{bmatrix} \mathrm{f} = 0$$

$$\mathrm{Af}_{\mathrm{norm}} = 0$$

$$F = N_2^T F_{norm} N_1$$

The normalized 8-point algorithm:
Step 3 – The singularity constraint

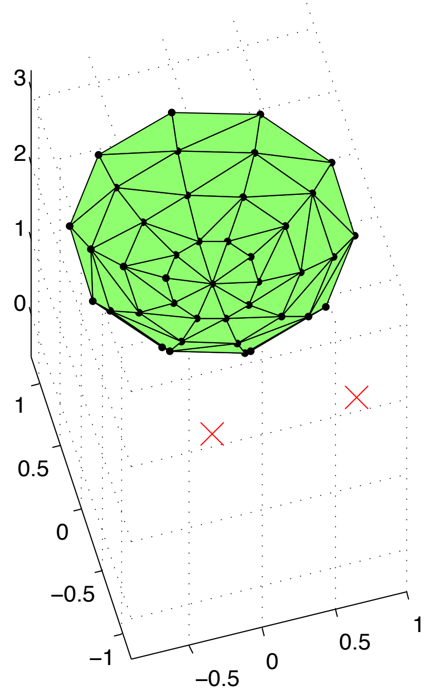
$$e'^T F = 0 \qquad Fe = 0 \qquad \det F = 0 \qquad \text{rank } F = 2$$

SVD from linearly computed F matrix (rank 3)

$$F = U \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} V^T = U_1 \sigma_1 V_1^T + U_2 \sigma_2 V_2^T + U_3 \sigma_3 V_3^T$$

Compute closest rank-2 approximation $\min \| F - F' \|_F$

$$F' = U \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & 0 \end{bmatrix} V^T = U_1 \sigma_1 V_1^T + U_2 \sigma_2 V_2^T$$



Evaluation procedure

1. Generate 3D points and cameras X_i, P, P'

2. Project 3D points into cameras $x_i = PX_i \quad x'_i = P'X_i$

3. Add noise to projections $\tilde{x}_i = x_i + \varepsilon \quad \tilde{x}'_i = x'_i + \varepsilon \quad \varepsilon \in N(0,\sigma)$

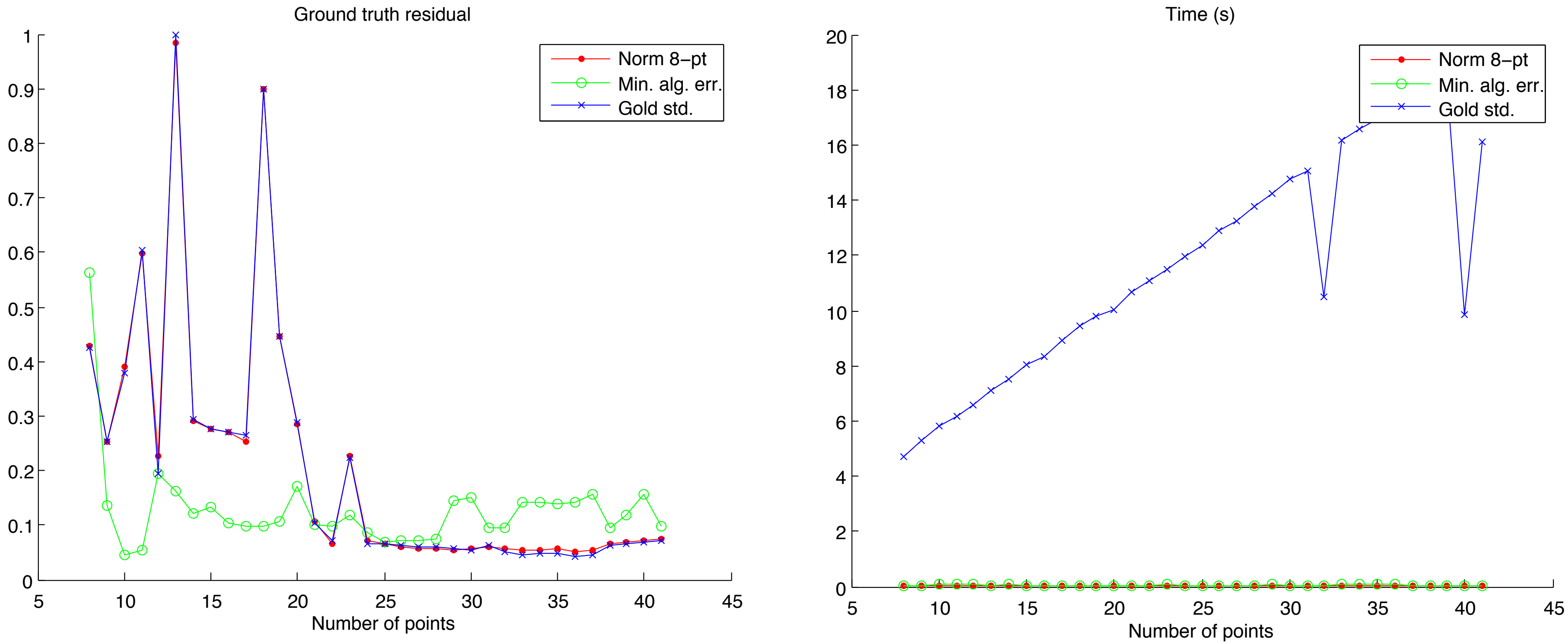
4. Estimate F $\tilde{x}_i \leftrightarrow \tilde{x}'_i \rightarrow F$

5. Compare F to ground-truth F_{gt}

$$e = 1 - \left(\frac{F \cdot F_{gt}}{\|F\| \cdot \|F_{gt}\|} \right)^2$$

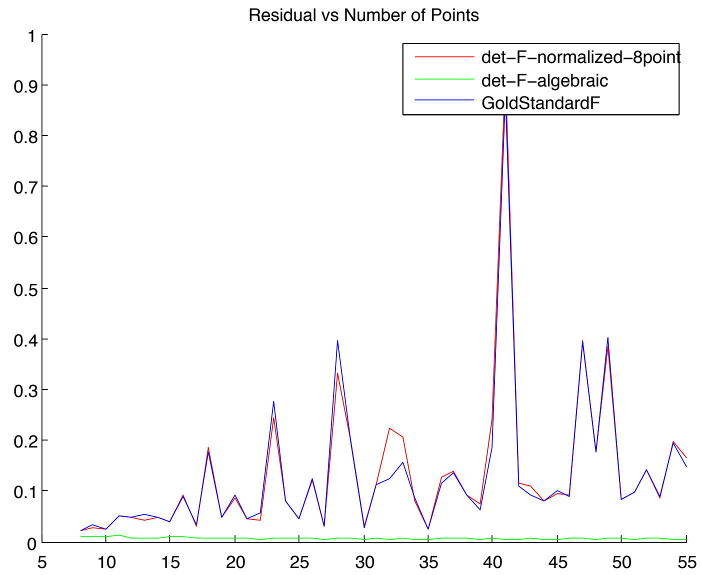
$$F_{gt} = [e']_{\times} \cdot P' \cdot P^{+}$$

Results

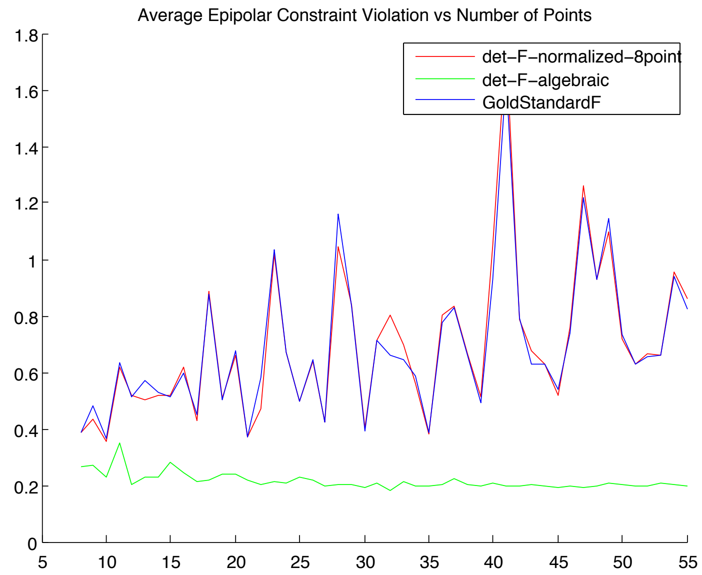


Ground truth residual
Norm 8−pt
Min. alg. err.
Gold std.
Number of points
Time (s)
Norm 8−pt
Min. alg. err.
Gold std.
Number of points

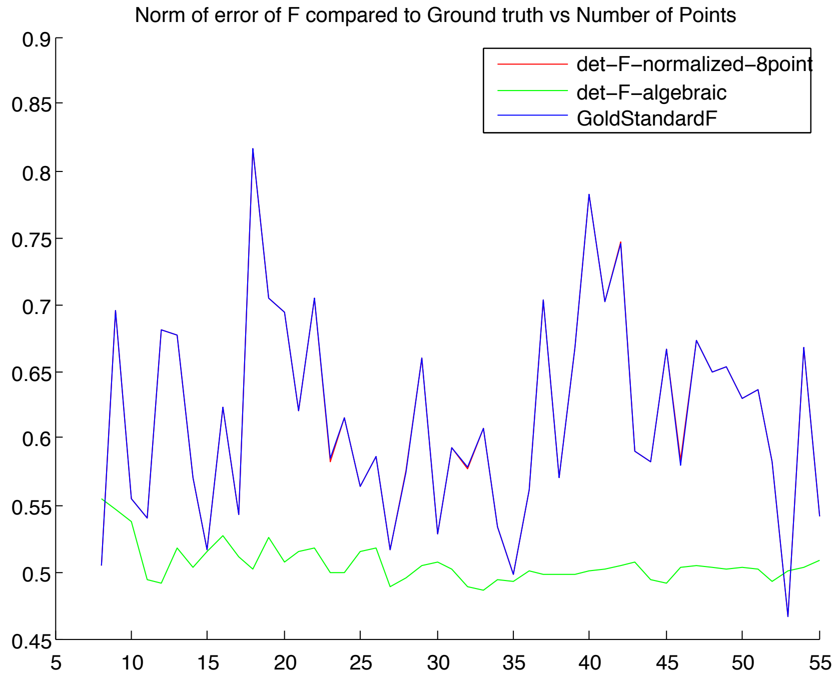
Results 2



Residual vs Number of Points
det−F−normalized−8point
det−F−algebraic
GoldStandardF



Average Epipolar Constraint Violation vs Number of Points
det−F−normalized−8point
det−F−algebraic
GoldStandardF



Norm of error of F compared to Ground truth vs Number of Points
det−F−normalized−8point
det−F−algebraic
GoldStandardF