Theoretical Bounds and Optimal Configurations for Multi-Pinhole SPECT

Peter Nillius and Mats Danielsson

***Abstract*—The pinhole geometry in SPECT has an inherent trade-off between resolution and sensitivity. High resolution requires a small aperture which on the other hand directly reduces the rate of detected photons. Recent systems overcome this to some extent by using multiple pinholes spread out around the imaging object, effectively increasing the sensitivity with a factor equal to the number of pinholes. The images of each pinhole must fit on the detector without overlap. This creates another trade-off between resolution, sensitivity and the field-of-view (FOV) of the system.**

The present work analytically analyzes the properties of the multi-pinhole SPECT geometry. Optimal configurations are identified and characterized. One of the main results is that there exists a theoretical upper bound for the sensitivity given the resolution and the FOV. This upper bound is proportional to the square of the resolution and inversely proportional to the square FOV diameter. This means that if we want to improve the resolution by a factor of ten, the sensitivity will go down a factor 100, unless we decrease the FOV an equal amount. The bound can not be broken even if the detector sphere is infinitely large.

One important parameter when designing a system is how close it will be to its theoretical bound. The closer to the bound the more extreme the system will be, in terms of size and number of pinholes. A moderate distance away from the bound the proportions of the system are more realistic.

I. Introduction

In vivo imaging of molecular mechanism using SPECT and PET is increasingly important in preclinical imaging. This has created the need for systems with higher resolution, especially in small animal imaging. PET resolution is limited to 1-3 mm by the range of positrons. SPECT has not this limitation but here higher resolution is traded for lower sensitivity. A number of systems has recently been developed that use multiple pinholes to achieve higher resolution while maintaining or even improving the sensitivity [1]–[3], [6].

Multi-pinhole systems has successfully achieved higher resolution and sensitivity. But what are the limits of this approach? The present work analytically analyses the fundamental properties of multi-pinhole SPECT. The goal is to learn how to optimally design such systems and to find the underlying limits of the technique.

II. Multi-pinhole geometry

The pinhole camera geometry has an inherent trade-off between resolution and sensitivity. To get a high resolution a small aperture is needed which in turn reduces the rate of detected photons, see Figure 1a. To increase sensitivity a collimator with multiple pinholes is used. These could be spread out on a sphere surrounding the object. The detectors lie on a larger sphere surrounding the collimator. The sensitivity is directly proportional to the number of pinholes, at least for a point which is visible to all pinhole cameras. The number of pinholes is limited by the fact that the images of all cameras have to fit on the surface of the detector sphere, see Figure 1b[1]. The size of the images on the other hand depends on the field of view and the camera geometry.

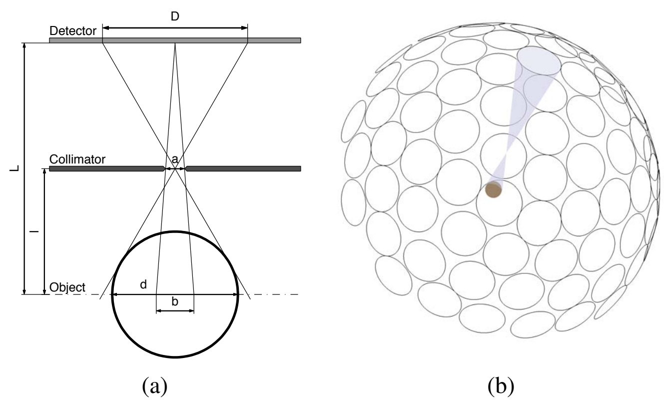

Fig. 1. Multi-pinhole camera geometry. (a) Single pinhole geometry. (b) In multi-pinhole cameras the images of the pinholes have to fit onto the detector sphere without overlap.

The geometric resolution, b is defined as the width of the beam at the detector from a point source in the middle of the field-of-view, divided by the magnification. This is the same as the width of the field of view for a single point on the detector as plotted in Figure 1a. The resolution and the pinhole aperture are related as

$$b = a\frac{L}{L-l} \tag{1}$$

and hence

$$a = b\frac{L-l}{L}. \tag{2}$$

The sensitivity of a single pinhole is the ratio of the pinhole area with the area of the collimator sphere. Approximating the spherical cap area of the pinhole with area of a disc we get the following expression for the single pinhole sensitivity.

$$S_{single} = \frac{\pi a^2}{16\pi l^2} = \frac{a^2}{16 l^2}. \tag{3}$$

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P. Nillius and M. Danielsson are with the Department of Physics, Royal Institute of Technology (KTH), SE-106 91 Stockholm, Sweden email: {nillius,md}@mi.physics.kth.se

[1] The images could be overlapping, as in Coded Aperture imaging, but this is beyond the scope of the paper.

Replacing the aperture with the resolution by inserting Equation (2) in (3) results in

$$S_{single} = \frac{b^2(L-l)^2}{16l^2L^2}. \tag{4}$$

This equation illustrates the resolution-sensitivity trade-off of the pinhole camera. A reduction in the resolution creates the same reduction squared in the sensitivity.

Let n be the number of pinholes. Then the system sensitivity S is

$$S = nS_{single} \tag{5}$$

The diameter, D, of the images on the detector is given by

$$D = d\frac{L-l}{\sqrt{l^2 - \frac{d^2}{4}}}. \tag{6}$$

The images should not overlap on the detector sphere. This results in the inequality

$$n\pi\frac{D^2}{4} \leq \rho 4\pi L^2, \tag{7}$$

where ρ is the best packing density of circles on a sphere. The packing density varies with the number of circles (pinholes), but as the number of pinholes grows this ratio converges to the best packing density of equal circles on $\mathcal{R}^2$, which is $\pi/\sqrt{12} \approx 0.9069$, [4].

Combining Equation (6) and (7) and isolating n leads to

$$n \leq \rho\frac{16L^2(l^2 - \frac{d^2}{4})}{d^2(L-l)^2}. \tag{8}$$

III. Theoretical Upper Bound

The fact that the images should not overlap on the detector creates a new trade-off between the sensitivity, the resolution and the field-of-view (FOV). In fact there exists an upper bound for the sensitivity S given the resolution b and the FOV d. We show that this upper bound is

$$S_{max} = \sup_{L>l>0} S = \rho\frac{b^2}{d^2}. \tag{9}$$

Figure 2 shows the upper bound for various resolutions and FOVs. For example, with a resolution of 1 mm it is possible to get an sensitivity 0.5-1%, if the FOV is small enough. To be able to achieve a resolution of 0.1 mm we need to go down to a 9 mm FOV for a maximum of 0.01% sensitivity.

The upper bound is reached only if the detector sphere is infinitely large. For detectors of finite size the upper bound is instead found to be

$$S_{max,finite} = \sup_{L>l>0} S = \rho\frac{b^2}{d^2}\left(1 - \frac{d^2}{4L^2}\right). \tag{10}$$

As can be seen, for $L >> d$ this bounds is very close to the infinite case. So, limiting the size of the detector sphere doesn't have a big impact on the theoretical bound of the sensitivity.

The finite size bound is reached when an infinite number of infinitely small pinholes are lying very close to the detector sphere. This also requires a detector with infinite resolution.

If we settle for performances below the upper bound there are more realistic configurations.

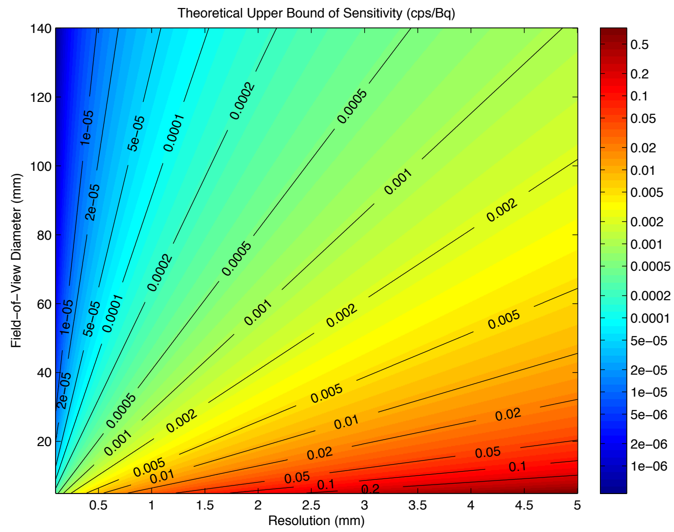


Fig. 2. Theoretical upper bound of the sensitivity as a function of resolution and field-of-view.

A. Derivation

This section derives the sensitivity upper bound.

To maximize sensitivity, we use the maximum number of pinholes allowed according to (8) such that

$$n = \rho\frac{16L^2(l^2 - \frac{d^2}{4})}{d^2(L-l)^2}. \tag{11}$$

Inserting Equation (11) into (5) and through some algebraic manipulation we obtain

$$S = \rho\frac{16L^2(l^2 - \frac{d^2}{4})}{d^2(L-l)^2}\frac{b^2(L-l)^2}{16l^2L^2} = ... = \rho\frac{b^2(1 - \frac{d^2}{4l^2})}{d^2} \tag{12}$$

From the expression in Equation (12) we see that for a fixed resolution b and FOV d the sensitivity is maximized when the distance to the collimator l is as large as possible and that

$$S \to \rho\frac{b^2}{d^2} \text{ when } l \to \infty, \tag{13}$$

which is exactly the upper bound in Equation (9).

As the system diameter L must be larger than l, the upper bound is reached with a system of infinite size. If L is finite we see that

$$S \to \rho\frac{b^2}{d^2}(1 - \frac{d^2}{4L^2}) \text{ when } l \to L, \tag{14}$$

which is the upper bound for multi-pinhole SPECT systems of finite size.

IV. Collimator Position and Number of Pinholes

For a given resolution and FOV, there exists many configurations that achieve the same sensitivity. These correspond to variations in the position of the detector layer, L and the number of pinholes, n. The position of the pinhole collimator, l is, as we will see, directly linked to the sensitivity of the system.

The maximum number pinholes is given by Equation (11). The number of pinholes can also be written as the total sensitivity divided by the sensitivity of a single pinhole, i.e.

$$n = \frac{S}{S_{single}} = \frac{16Sl^2L^2}{b^2(L-l)^2} \tag{15}$$

By combining Equations (11) and (15) and solving for the collimator position we get

$$l = \frac{d}{2}\frac{1}{\sqrt{1-\frac{Sd^2}{\rho b^2}}}. \tag{16}$$

In this expression we identify the sensitivity upper bound from (9) and rewrite it as

$$l = \frac{d}{2}\frac{1}{\sqrt{1-\frac{S}{S_{max}}}}. \tag{17}$$

Consequently, the position of the pinhole collimator is directly linked to the sensitivity of the system. It has been reported that the collimator should be as close as possible to the object [5]. This is more or less true for $S << S_{max}$, but as we get closer to the upper bound the collimator layer has to be moved further back. For example, $S = 0.5S_{max}$ puts the collimator at $l = \sqrt{2}d$, $S = 0.99S_{max}$ at $l = 5d$ and of course $S = S_{max}$ will put the collimator at infinity. See Figure 3a for a plot on how the collimator placement varies with resolution and sensitivity.

In a similar fashion we can compute the minimum number of pinholes, n_{min}. Expressed in terms of the upper bound it is found to be

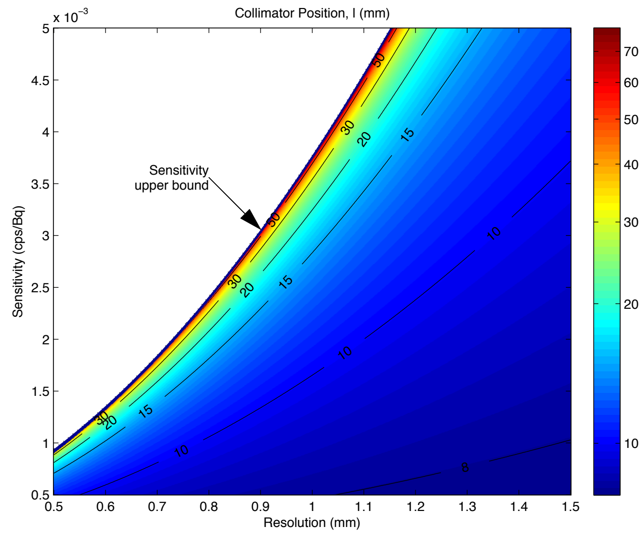
$$n_{min} = \frac{4\rho}{\frac{\alpha_{max}}{\alpha} - 1}. \tag{18}$$

We can see that the number of pinholes increases as the desired sensitivity gets closer to the upper bound. Figure 3b shows n_{min} for different resolutions and efficiencies.

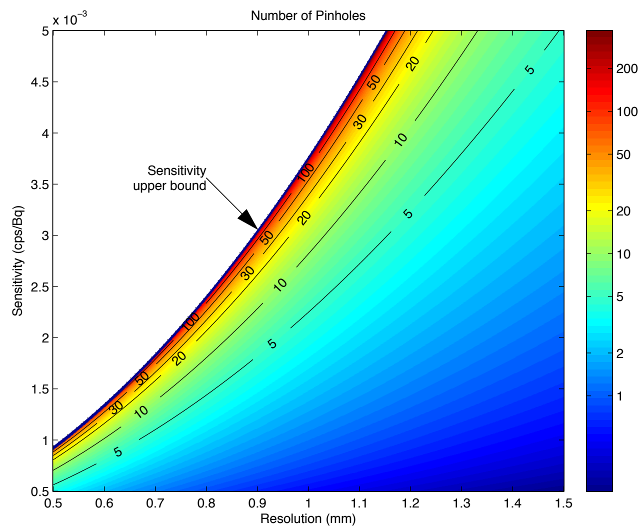
V. Conclusions

SPECT systems with multiple pinholes has successfully been used to increase resolution and sensitivity. We show that there is a theoretical upper bound beyond which the performance of multi-pinhole SPECT cannot be pushed, even if the detector would be infinitely large. This bound is in fact reached with a detector of infinite size. Limiting the size of the detector lowers the upper bound, but only marginally when detector size is much larger than the field-of-view, which is normally the case.

Moreover, we have designed simple mathematical tools to visualize the trade-offs between resolution, sensitivity and field-of-view. These tools can be used as a guide to find optimal configurations when designing a system with a desired performance. One important parameter when designing a system is how close it will be to its theoretical bound. The closer to the bound the more extreme the system will be, in terms of size and number of pinholes. A moderate distance away from the bound the proportions of the system are more realistic.



(a)



(b)

Fig. 3. Contour plots of (a) collimator placement and (b) number of pinholes required to achieve a given resolution and sensitivity. As we get closer to the theoretical bound (dotted line) the collimator is pushed further back and the number of pinhole goes towards infinity. The field-of-view in this example is 15 mm.

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