# Game Theory Examples

Adopted from Sarit Kraus

”Strategic Negotiation in

Multi-Agent Environments”

# Strategic Negotiation Model

- A set of agents involved in a turn-taking negotiation (offering solution)
  - Offer ! Yes/no/opt out
- Assumptions
  1. Rational agents
  2. Avoid opting out
  3. Commitments are kept
  4. No long term commitments
  5. Common belief (1-4)

# Utility Functions

- Fixed loss/gain per period
  - $U^i(o,t)=U^i(o,0) + t * C$
- Time constant discount
  - $U^i(o,t) = \delta^t_i * U^i(o,0)$, $\delta$ 2 [0;1]
- Financial interest rate model
  - $U^i(o,t) = 1/(1+r)U^i(o,0) + C\ (1+r)/r *(1-1/(1+r)^t)$
- Finite horison models
  - $U^i(o,t) = U^i(o,0)*(1-t/N) – t*C$
- U(o,0) initial value, C = cost /loss per period

# Game Solutions

- Nash Equilibrium
  - No agent can benefit from deviation given actions of others
- Sub-game perfect equilibrium
  - At each stage a Nash equilibrium is the solution
- Sequential equilibrium
  - Given incomplete information, a Nash equilibrium is determined based on belief

# Case 1: Negotiation about resource allocation

- Two agents negotiate about a common resource
- Ex: Two robots on Mars
  – NASA and ESA.
- Bilateral negotiation
- Two agents
  – Attached agent – holder of the resource
  – Waiting agent – Waiting to gain access to resource
- Fixed resources, say M
- Solution: $s_a + s_w = M$

# Motivation

- Future negotiation
  - Fear of future loss
- Waiting agent threat
  - The other agent might destroy resource
- Costless process
  - The actual negotiation is inexpensive

# Utility Function Properties

- A0: Disagreement the worst outcome
  - For x 2 { |S [ OPT| £ T}: $U^i(\text{disagree}) < U^i(x)$
- A1: The resource is valuable
  - For t2T, r,s2S: r>s ) $U^i(r,t) > U^i(s,t)$
  - Maximize access to resource
- A2: Cost benefit over time
  - For $t_1,t_2$2 T, $t_1<t_2$:
  - $U^W(s,t_1)<U^W(s,t_2)$ and $U^A(s,t_1)>U^A(s,t_2)$

# Utility Function Properties II

- A3: Agreements cost over time
  - i2 {A,W}, 8 $t_1$, $t_2$ 2 T, $s_1$,$s_2$ 2 S, $c_i$ )
  - $U^i(s_1,t_1) > U^i(s_2,t_2)$ iff
  - $(s_i+c_i\ t_1)$ ¸ $(s_{2i} + c_i\ t_2)$
- A4: Cost of opting out over time
  - 8 t2 T:
  - $U^W(opt,t)>U^W(opt,t+1)$ & $U^A(opt,t)<U^A(opt,t+1)$

# Range of agreements

- A5: 8 t2T
  - If $Possible^{t+1} \neq \emptyset$ ) $Possible^{t} \neq \emptyset$
  - If $Possible^{t+1} \neq \emptyset$ )
    - $U^{W}(s^{W,t},t)$ ¸ $U^{W}(s^{W,t+1},t+1)$
    - $U^{W}(opt,t)$ ¸ $U^{W}(s^{w,t+1},t+1)$
    - $U^{A}(s^{w,t+1},t+1)$ ¸ $U^{A}(s^{w,t},t)$
  - If $Possible^{t} \neq \emptyset$ )
    - $U^{A}(s^{w,t},t)$ ¸ $U^{A}(opt,t+1)$

# Agreements are possible

- A6: Possible agreements
  - $Possible^0 \neq \emptyset$ and $Possible^1 \neq \emptyset$
  - During the first two periods there are outcomes better than opting out

# Example

- NASA & ESA have two robots on Mars
  - Joint mission
  - NASA damaged antenna (1 day repair)
    - Use of backup line is expensive
  - Sharing of ESA antenna
  - ESA is using some NASA equipment in its experiments

# Example

- ESA is earning $5000 / minute of experimentation
- NASA is loosing $3000 / minute of backup line usage
- Per minute of shared usage $1000 is earned by each group
- NASA total gain is $550000, loss of $1000 / minute of negotiation
- ESA cost of US equipment is $100000 if the opt out.
- Constant gain/loss example with a finite horison

# Formally

- $U^e(s,t) = 1000\ s_e + 5000\ t$
- $U^e(opt_n,t)=5000\ t$
- $U^e(opt_e,t) = 5000\ t – 100000$
- $U^n(s,t) = 1000\ s_n – 3000\ t$
- $U^n(opt_n,t) = 550000 – 1000\ t$
- $U_n(opt_e,t) = -1000\ t$
- M = 1440 (1 day = 1440 minutes)

# Formally

- $c_e = 5$, $c_n = -3$
- $s^{n,t} = (890-2t, 550 + 2t)$
- $U^n(s^{n,t},t) = (550-t) * 10^3$
- $U^e(s^{n,t},t) = (890 + 3t) * 10^3$
- An agreement is achieved in the second period (888,552)
  - ESA earn over time, but opting out might cause a loss, NASA losses over time, but waiting might not result in getting access

# What could a mediator do?

- Earliest solution that is ”popular”
- Nash product is an approach
  - Maximization of
  - $(U^A(x,0)–U^A(opt_a,0))$£ $(U^W(x,0)–U^W(opt_w,0))$
- As loss by W is larger than gain by A, W will prefer an early solution (step 1), while A prefers a late solution. Mediator solution will be a solution is step 1. Asymmetries cause a preference for mediation.

# Case 2: Negotiation about Task Distribution

- Several agents to share a common task
- M jobs to be completed
- Time before an agreement = loss
- Agreement $s_1 + s_2 = M$

# Attributes of Utility Function

- A1: Actions are costly
  - $t \in T,\ r,s \in S$:
    $r_i > s_i \Rightarrow U^i(r,t) < U^i(s,t)$
- A2: Time is valuable
  - $t_1,t_2 \in T,\ s \in S$:
    $t_1 < t_2 \Rightarrow U^i(s,t_1) > U^i(s,t_2)$
- A3: Agreements cost over time
  - Each agents has a cost $c_i>0$ such that
  - $\forall\ r,s \in S$ Æ $t_1,t_2 \in T$:
    $U^i(s,t_2) > U^i(r,t_2)$ iff $(s_i - c_i\ t_1) \geq (r_i - c_i\ t_2)$

# Attributes of Utility Function

- A4: Opting out costs more over time
  - 8 $t_1,t_2$ 2 T:
    $t_1<t_2$ ) $U^i(opt,t_1)<U^i(opt,t_2)$
- A5: Agreements vs Opting Out
  - 8 t 2 T:
    $U^i(s,t)>U^i(opt,t)$Æt ¸ 1 ) $U^i(s,t-1)>U^i(opt,t-1)$
- A6: Time period when an agreement if not possible
  - 9 t2 T: $Possible^t = \emptyset$, $t_m$ = min t, $Possible^0 \neq \emptyset$

# Solution

- A solution is always reached
- If agent i is to propose a solution at $t_m$-1, it will suggest its sub-game perfect equil. The other will accept it.

# Ex: delivery of newsletters

- Two news agencies N1, N2
- Two delivery agents D1, D2
- N1 ! D1 is paid $200,
  N2 ! D2 is paid $225
- Delivery cost $1, A period of wait is $1
- M subscribers of N1 and N2
- If agreement N1 ! D1 = $170,
  N2 ! D2 = $200,
  Still $1 / call, $2 / period of wait

# Formally

- $U^1(opt,t) = 200 – M – t$ Æ
  $U^1(s,t) = 170 – s_1 – 2t$
- $U^2(opt,t) = 225 – M – t$ Æ
  $U^2(s,t) = 200 – s_2 – 2t$
- Ex: M=100 ) :
  - $s^{2,t}=(69-t,31+t)$,
    $s^{1,t}=(26+t,74-t)$,
    $t_m = 22$ ) agreement (46,54)