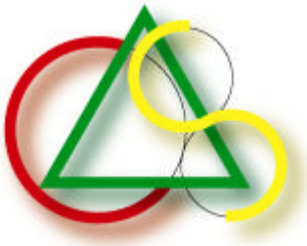

Game Theory

Henrik I Christensen

Motivation

- Robotics is increasingly seen as a set of independent ’processes’ that compete and collaborate to achieve specific (myopic) objectives.
- Game theory offers a theoretical basis for analysis and design of the interaction between such players. Processes can here be interpreted as ’behaviours’ or individual robots.

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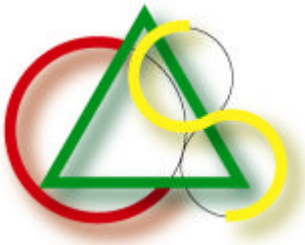
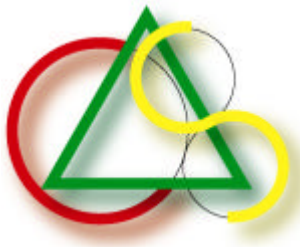


Literature

1. M. J. Osbourne & A. Rubenstein, *"A Course in Game Theory"*, MIT Press, 1994
2. D. Fudenberg & J. Tirole, *"Game Theory"*, MIT Press, 1998
3. H. W. Kuhn, *"Classics in Game Theory"*, Princeton University Press, 1997
4. G. William Flake, *"The Computational Beauty of Nature"*, MIT Press, 1999

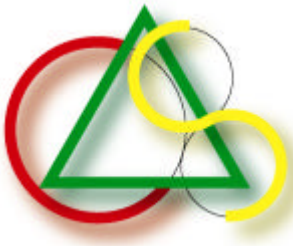
Game Theory

- Bag of analytical tools to understand decision making and interaction
- Agents pursue exogenous objectives (they are rational)
- Take into account knowledge and expectation of other agents (reason strategically)



(Some) Types of Games

- Strategic games
- Extensive games with perfect knowledge
- Extensive games with incomplete knowl
- Coalition games



(non) Cooperative Games

- Basic Entity: player/agent
- Distinction between individual players and groups of players
  - Simple games (non-cooperative games)
  - Coalition games (cooperative games)
- Recent research has emphasized non-cooperative games – The theory is pretty!

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Strategic & Extensive Games

- Strategic Games
  - Each player chooses his plans of actions
  - All moves are simultaneous
- Extensive Games
  - Turn taking between players
  - Plan of actions are adaptive

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Perfect vs Imperfect Information

- Perfect Information
  - All participants share information about all actions
- Imperfect information
  - Each player has only partial information about the moves of other players

Rational Behaviour

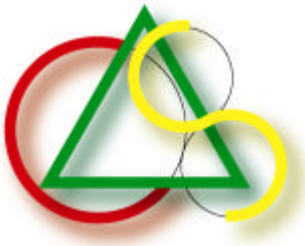
- Rational:
  - Aware of alternatives, expectation about unknowns, has preferences and chooses actions deliberately to optimise some process
- A Model
  - **A** a set of *actions*
  - **C** a set of possible *consequences*
  - A *consequence function* **g**:A ! C
  - A *preference relation* º over C
  - Optional a *utility function* **U**:C ! $\mathcal{R}$ which defines a preference relationship x º y iff U(x) ¸ U(y)

Rational decision making

- A player decides on an a* action from the feasible set B µ A
  - Optimal g(a*) º g(a) for all a 2 B or
  - Solves max $_{a2B}$ U(g(a))
- Uncertainty may arise from
  - Uncertainty about object parameter about env.
  - Imperfect info about prior events in the game
  - Uncertainty about actions of other players
  - Uncertainty about reasoning of other players

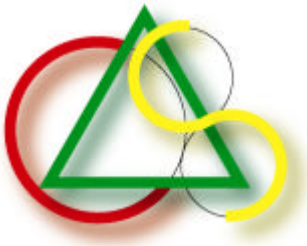
Decision making under uncertainty

- Based on basis by Neumann & Morgenstein (1944)
- Consequence function g is stochastic
  - I.e. For a2A the function g(a) is a lottery
  - Maximises the expected value
  - Alternatively: A state space is available Ω, and a probability measure over Ω, g:A£Ω ! C with a utility function U:C ! R, then u(g(a,ω)) is maximized



Strategic Game

- Definition of a strategic game
  - A finite set of players **N**
  - Each player has an action set $\mathbf{A_i}$
  - A preference relation for each player $\succ_i$ on $A = \times_{j \in N} A_j$
  - If $|A_i|$ is *finite* the game is *finite*
  - *Potentially* a utility function $u_i: A \rightarrow R$
  - A strategic game is denoted $\langle N,(A_i),(u_i)\rangle$ or $\langle N,(A_i),(\succ_i)\rangle$



Nash Equilibrium

- **Definition**
  - Given $<N,(A_i),(\succ_i)>$
  - A Nash equilibrium is a profile $a^* \in A$ that satisfies

$$(a^*_{-i}, a^*_i) \succ (a^*_{-i}, a_i) \quad \forall a_i \in A_i$$

for all players $i \in N$

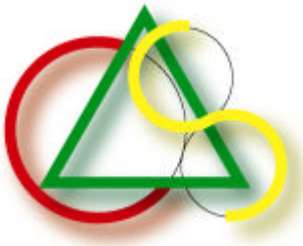
Nash Equilibrium

- Alternatively
  - Define $B_i(a_{-i})$ $8a_{-i}2A_{-i}$ as **best action** for player *i* given a_{-i}:

    $B_i(a_{-i}) = \{a_i 2 A_i : (a_{-i}, a_i) \, º \, (a_{-i}, a'_i) \; 8a'_i 2 A_i\}$

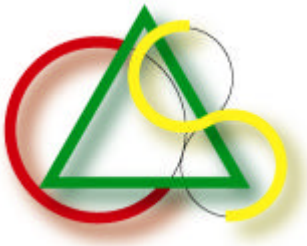
  - **B** is the best response function of player *i*
  - A Nash Equilibrium is a profile a* for which

    $a^*_i 2 B_i(a^*_{-i})$ 8i2N



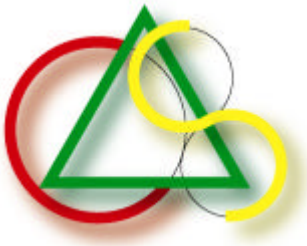
Example: Bach or Stravinsky

	Bach	Stravinsky
Bach	2,1	0,0
Stravinsky	0,0	1,2



Example: Prisoner's Dilemma

	Don't Confess	Confess
Don't Confess	0,0	-4,1
Confess	1,-4	-3,-3

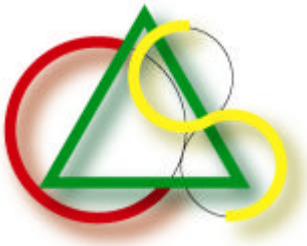


Matching Pennies

Example of a zero-sum game

	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

No Nash Equilibrium



Coffee making

	Cooperate	Defect
Cooperate	(CC,CC) Reward from mutual coop.	(CD,DC) Suckers payoff and temptation to defect
Defect	(DC,CD) Temptation to detect, and Sucker's payoff	(DD,DD) Punishment for mutual defection

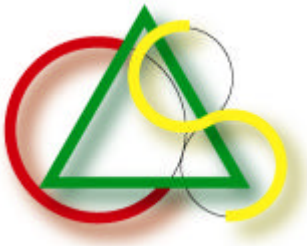
Payoff priorities

DC: Get to drink and the others brew it

CC: Drink coffee and make fair share

CD: Drink coffee but are exploited by others

DD: No one get coffee



Variations

- Chicken
  - DC > CC > CD > DD
- Stag Hunt
  - CC > DC > DD > CD
  - {Football}
- Prisoners Dilemma
  - DC > CC > DD > CD

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Extended Games Appetizer

- Example: Iterated Prisoners Dilemma

Strategies:

Always defect – ALL-D

Always Cooperate – ALL-C

Random coop/defect – RAND

Reward:

DC = 5p (temptation)

CC = 3p (cooperation)

DD = 1p (mutual defection)

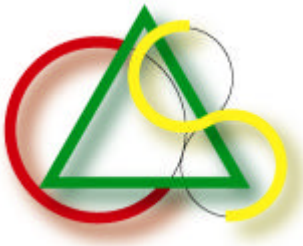
CD = 0p (sucker's payoff)

	ALL-C	ALL-D	RAND	Average
ALL-C	3.0	1.5	0.0	1.5
ALL-D	4.0	2.0	0.5	2.16
RAND	5.0	3.0	1.0	3.0

TFT-Strategy

- Tit-for-tat strategy ~4 lines of code
- Cooperate 1 round
- Do what opponent did in previous round

- Highly effective strategy



Ecology & Spatial Wars

- Closed environment ~ limited resources
- Describes by Population (P_i) & Score (S_i)
- Update

$$P_i(t+1) = \frac{P_i(t) \times S_i(t)}{\sum_{j=1}^{N} P_j(t) \times S_j(t)}$$

$$S_i(t) = \sum_{k=1}^{N} P_k(t) \times R_{ik}(t)$$

- R_{ij} Score table a la IPD

Spatial War

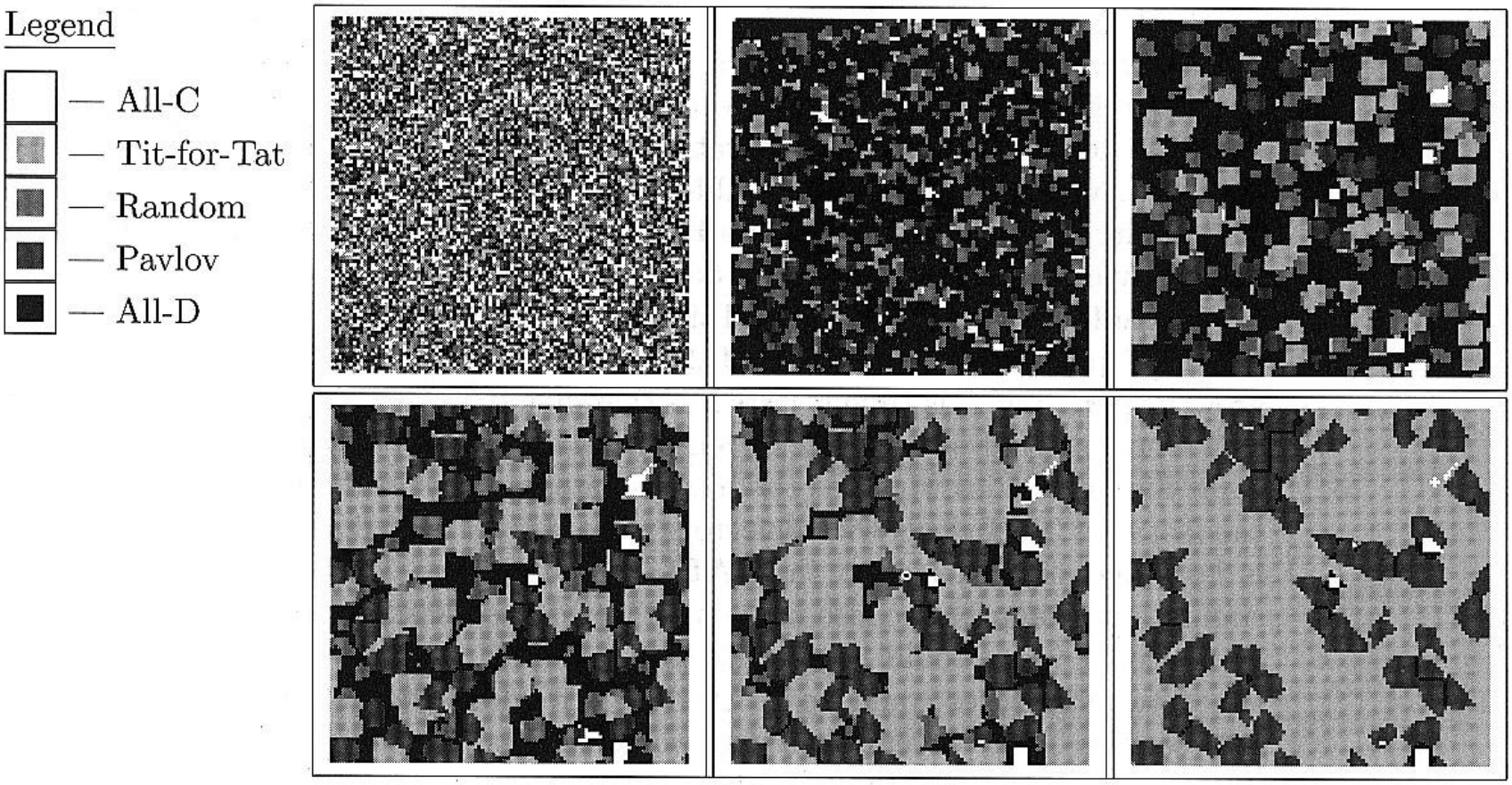
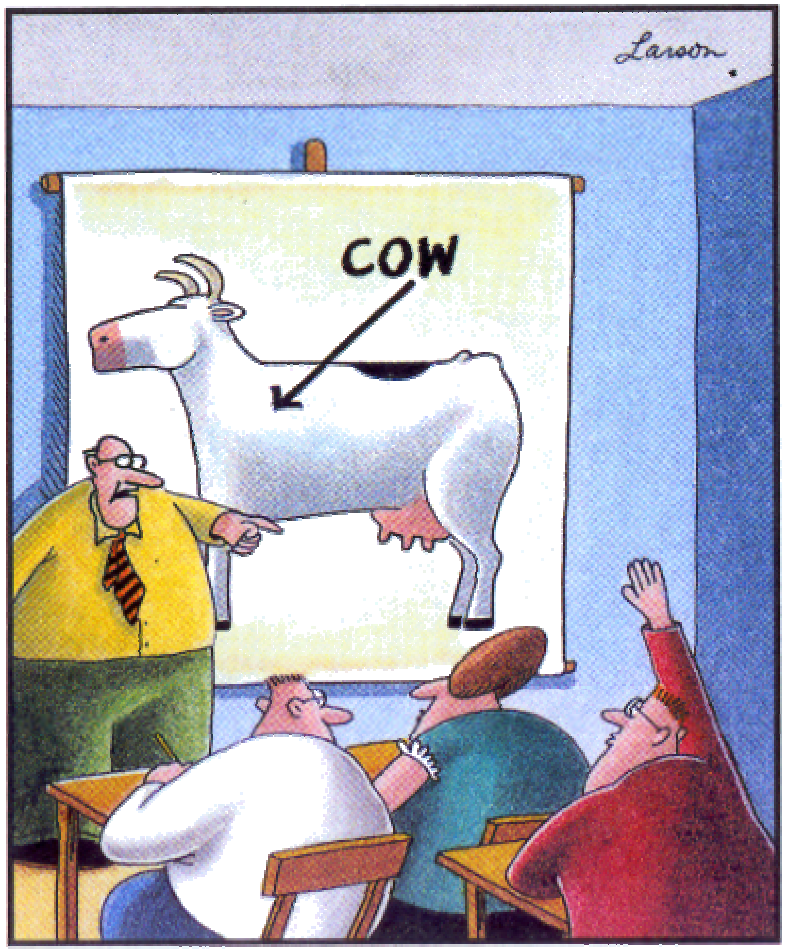


Figure 17.4 Competition in the spatial iterated Prisoner's Dilemma without noise

Additional Topics

- Extended Games
- Games with imperfect knowledge
- Examples from Control and Ecology
- Utilization of GT for Robotics
  - Behaviour Coordination
  - Multi-Agent Coordination
- Is this a worthwhile use of our time?
- Emphasis?

Questions/Discussion



"Yes ... I believe there's a question in the back."