1 Notes about the initial value problem (Begynnelsevärdeproblem)

Problem description:

$$\begin{aligned} \frac{dy}{dt} &= f(t, y) \quad \text{for } t \in [t_{start}, t_{end}] \\ y(t_{start}) &= y_0 \quad \text{initial value} \end{aligned}$$

Calculate numerically the function $y(t)$ by advancing in t with stepsize $h = \frac{t_{end} - t_{start}}{N-1}$ (N the number of interval points) and calculating an approximation $y_i \sim y(t_i)$.

1.1 Euler method(forward, explicit)

$$y_{i+1} = y_i + h * f(t_i, y_i)$$

The global error is of order $O(h)$.

1.2 Runge-Kutta method

Runge-Kutta methods evaluate the function in several points.

1.2.1 RK2

$$\begin{aligned} y_{i+1} &= y_i + \frac{h}{2} * (f_1 + f_2) \\ f_1 &= f(t_i, y_i) \\ f_2 &= f(t_i + h, y_i + h * f_1) \end{aligned}$$

The global error is of order $O(h^2)$.

1.2.2 RK4

$$\begin{aligned} y_{i+1} &= y_i + \frac{h}{6} * (f_1 + 2 * f_2 + 2 * f_3 + f_4) \\ f_1 &= f(t_i, y_i) \\ f_2 &= f(t_i + \frac{h}{2}, y_i + \frac{hf_1}{2}) \\ f_3 &= f(t_i + \frac{h}{2}, y_i + \frac{hf_2}{2}) \\ f_4 &= f(t_i + h, y_i + hf_3) \end{aligned}$$

The global error is of order $O(h^4)$.

1.3 ode23, ode45

ode23 and ode45 are matlab-defined functions.

$$[T, Y] = \text{ode23}('func', [0 \quad t_{t_{slut}}], y_0)$$

The default value for the relative tolerance is set to 10^{-3}. If one wants to change it set $tol =$ odeset$('RelTol', 1e - 8)$

$$[T, Y] \quad = \quad \text{ode23}('func', [0 \quad t_{t_{slut}}], y_0, tol)$$