On the other side it is also possible to see that (A.2) is equivalent to

$$\begin{array}{rl} \texttt{minimize} & c^T x \\ \texttt{subject to} & Ax = b \\ & x \geq 0 \end{array}$$

where we assume that all variables are sign-constrained.

In fact:

1. Any inequality constraint $ax \geq b_i$ can be transformed in an equality constraint by adding a new *surplus* variable s_i (also called *slack variable*). The constraint is then replaced by the pair $ax - s_i = b_i$, $s_i \geq 0$.

2. For any unconstrained variable x_j, two variables x_j^+, x_j^- are introduced that are sign constrained ($x_j^+ \geq 0$ and $x_j^- \geq 0$); any occurrence of x_j is replaced by $x_j^+ - x_j^-$.

A linear program can also be used to formulate maximization problems such as,

$$\begin{array}{rl} \texttt{maximize} & c^T x \\ \texttt{subject to} & Ax \leq b \\ & x \geq 0. \end{array} \tag{A.3}$$

A correspondence between maximization and minimization problems can be easily established. In fact:

1. Any maximization problem can be transformed into a minimization one by considering the objective function $-c^T x$.

2. Any inequality constraint $ax \leq b_i$ can be transformed into an equality constraint by adding a new slack variable s_i. The constraint is then replaced by the pair $ax + s_i = b_i$, $s_i \geq 0$.

Given a minimization linear program, called *primal*, it is possible to define a maximization linear program, called *dual*, as follows: for each constraint in the primal program, there is a variable in the dual; for each variable in the primal program there is a constraint in the dual. Namely, we have: