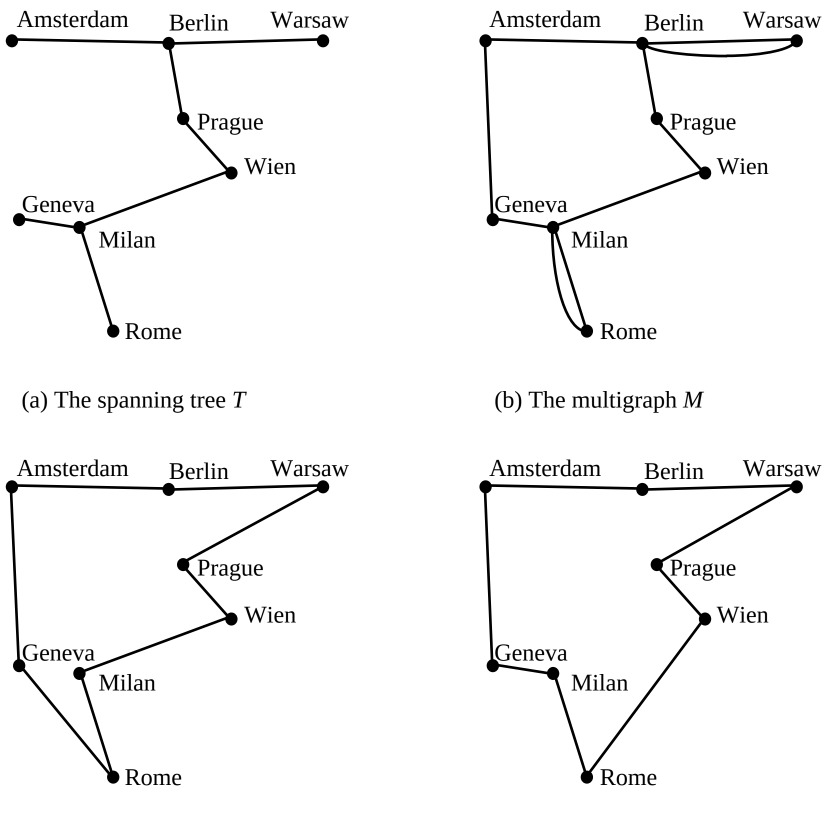

(a) The spanning tree T

(b) The multigraph M

(c) The approximate tour I

(d) The optimal tour I^*

Figure 3.3
A sample application of Christofides' algorithm

◀ Example 3.3

For any positive integer n, let us consider the instance G_n of MINIMUM METRIC TRAVELING SALESPERSON shown in Fig. 3.4(a), where all distances that are not explicitly specified must be assumed to be computed according to the Euclidean distance. It is easy to see that one possible minimum spanning tree is the tour shown in the figure without the edge (a_1, a_{n+1}). If Christofides' algorithm chooses this spanning tree, then the approximate tour shown in Fig. 3.4(a) results: note that this tour has measure $3n+2\varepsilon$. On the contrary, the optimal tour is shown in Fig. 3.4(b) and has measure $2n+1+4\varepsilon$. As n grows to infinity, the ratio between these two measures approaches the bound $3/2$.

Until now Christofides' algorithm is the best known approximation algorithm for MINIMUM METRIC TRAVELING SALESPERSON. While in the Euclidean case arbitrarily good polynomial-time approximation algorithms can be found (see Bibliographical notes), no polynomial-time approximation algorithm with a better guaranteed performance ratio is known for MINIMUM METRIC TRAVELING SALESPERSON, neither it is known whether the existence of any such algorithm would imply $\mathrm{P} = \mathrm{NP}$.